

The University of Chicago

STATISTICAL DETERMINATION
OF COST, WITH SPECIAL REFERENCE
TO MARGINAL COST

A PART OF A DISSERTATION SUBMITTED TO
THE FACULTY OF THE SCHOOL OF BUSINESS
IN CANDIDACY FOR THE DEGREE OF DOCTOR
OF PHILOSOPHY

1936

By
JOEL DEAN

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PREFACE

The principal objectives of this investigation are, first, to develop practical methods for determining the behavior patterns of a firm's average and marginal cost and, second, to explore by means of these methods the actual cost phenomena of two sample enterprises, one with short-run and the other with long-run cost variations. This statistical study should be of interest to business men, to economists, to statisticians, and to cost accountants.

To business men it should be useful because these findings constitute a new type of managerial information which leads to more effective cost control and more profitable price policy. Quantitative knowledge of the effect of various operating conditions (such as the rate of output, the size of production order, new styles, and the like) upon average and total manufacturing cost has great managerial value since it is needed, first, for constructing accurately a comprehensive flexible budget; second, for establishing adjusted standard costs; and, third, for estimating marginal cost.

Budgets are most effective when they can be flexibly modified to correspond to reasonable expectations of the cost which will result from the particular set of operating conditions prevailing in the budget period. Quantitative knowledge of the probable effect upon cost of these conditions makes such budget modifications more accurate and thus renders the budget more useful for prediction and for control. Similarly, cost standards which can be adjusted, on the basis of known cost relationships, so as to allow for operating conditions over which the factory management has no control and for which it should therefore not be held responsible, are fairer and more effective as control devices. Probably the most important benefit derived from analysis of cost variation is knowledge of marginal cost. Estimates of incremental cost are not provided by ordinary accounting reports and cannot be made accurately without quantitative knowledge of the effect of volume changes, and changes in other operating conditions, upon total cost. Accurate knowledge of marginal cost is of great importance in the rational formulation of business policies since a large proportion

of decisions should be made by comparing the marginal costs of alternative methods or by balancing the added cost caused by a contemplated policy against the resulting additional revenue.

Economists will find this study stimulating, first, because it presents an empirical determination of cost curves which play a crucial rôle in modern price theory; second, because it develops a method for providing the entrepreneur with knowledge of his marginal cost—a type of information which economists have erroneously assumed he possessed; and, third, because it indicates certain areas of discrepancy between neoclassical cost theory and reality.

Statisticians will be interested in such a study because it shows the type of technical difficulties to be encountered and the kind of useful results to be obtained by applying familiar methods to a neglected field of investigation.

Cost accountants should be cognizant of the procedures developed in this study because these methods of analyzing and interpreting accounting records provide a kind of information which accountants should, but commonly do not, submit,

The failure of accountants to provide business men with information regarding marginal cost has encouraged rigid prices in certain monopoly areas and has consequently been one important influence toward prolonging the depression. Lacking a knowledge of his marginal cost, the monopolist has taken the accountant's average cost as his price guide. With curtailed demand during the depression this average cost rose, and consequently the advice of the accountant was usually sternly against price reductions, even though lower labor and material prices may have reduced marginal costs. This widespread delusion of rigid or rising costs has been an important cause for price rigidities in areas of imperfect competition. Thus, knowledge of marginal cost may be looked upon as a factor making for more general price flexibility and for greater business stability.

In conclusion, it should be emphasized that the cost functions obtained herein are not presumed to be typical, but merely illustrative of the results to be obtained from the simple, practical techniques developed in this study. Subsequent investigations, with better data and more refined procedures, will doubtless produce more reliable and significant results. The chief functions of this study are to point out

the obstacles to be overcome and to suggest a feasible method for providing management with a type of information which is really relevant for cost control and for price policy.

It is a pleasure to acknowledge with gratitude the important assistance rendered by many persons. More than to anyone else I am indebted to Professor T. O. Yntema of the University of Chicago who, by his encouragement and help at all stages of this investigation, has contributed much to whatever merit it possesses. To Professor James E. Moffat of Indiana University I owe much for his generous encouragement. I am also particularly indebted to Professor Garfield Cox, Professor Henry Schultz, and Professor Jacob Viner, who have helped me considerably both directly and indirectly. Professor C. Lawrence Christenson and Professor William Cleveland of Indiana University; Professor E. A. Duddy, Professor Martin Freeman, Professor Willard Graham, and Professor Samuel Nerlove of the University of Chicago; Mr. Benjamin Caplan and Mr. Sune Carlsen have all read portions of the manuscript and made valuable suggestions. To Mrs. Samuel Yellen, Miss Emma Peterson, and Mr. Rondal Huffman I am indebted for valuable technical assistance, and to the officers of the two firms whose data were used in this study I am grateful for their generous co-operation.

Since I have been unable to satisfy all my critics, I must assume full responsibility for all inaccuracies and inadequacies of this analysis.

JOEL DEAN

INDIANA UNIVERSITY

November, 1936

TABLE OF CONTENTS

CHAPTER	PAGE
I. SUMMARY AND CONCLUSIONS	I
II. SURVEY OF PREVIOUS STUDIES OF COST BEHAVIOR	13
III. METHODS OF PREPARING COST DATA FOR ANALYSIS	24
IV. METHODS OF ANALYZING COST DATA	38
V. RELIABILITY OF FINDINGS	47
VI. BEHAVIOR OF AVERAGE COMBINED COST	56
VII. BEHAVIOR OF MARGINAL COMBINED COST	74
VIII. PRACTICAL APPLICATIONS OF FINDINGS	98
APPENDIX	
A. BEHAVIOR OF COMPONENTS OF AVERAGE COMBINED COST	109
B. BEHAVIOR OF COMPONENTS OF MARGINAL COMBINED COST	130

CHAPTER I

SUMMARY AND CONCLUSIONS

OBJECTIVES

The principal objectives of this study are, first, to investigate the potentialities and limitations of various statistical methods for analyzing cost behavior, with a view to developing practical procedures for determining the short-period behavior of average and marginal cost; and, second, to examine by means of these methods the behavior of average and marginal cost for a particular manufacturing establishment.

By average cost is meant the cost per unit of output, i.e., the total cost for the entire period divided by the number of units produced during that period. And by marginal cost (also called differential and incremental cost) is meant that addition to the total cost caused by the production of an additional unit of product, i.e., that particular increase in total cost which is associated with a unit increase in output. The short period may be defined as a time span too brief to permit alterations in the scale or character of the fixed equipment. For the purposes of this study accounting periods of two weeks' duration were used.

These objectives were deemed worth while because, in the first place, a knowledge of the short-period behavior of average and marginal cost is of considerable utility in the management of any business and, in the second place, few systematic procedures for securing this knowledge have been developed.

An understanding of the behavior of average cost may be useful to the business executive on several accounts: (1) it will aid in establishing flexible cost standards which can be used for controlling cost;¹

¹ A more debatable use for flexible cost standards is in evaluating inventories. For this purpose they share with fixed standard cost the advantages of timeliness (since they do not await the closings of the books) and of economy (since detailed records for particular lots of product are not required—at least, not for inventory purposes). At the same time flexible cost standards exhibit some of the advantages of historic cost in that they approximate more nearly the actual costs peculiar to the operating conditions of the period.

(2) it will enable the management to institute programs of cost reduction based upon more accurate knowledge of the causes underlying cost behavior; and (3) it will assist in computing estimates of cost and of total profit under predicted conditions of future operation.

The superiority of adjustable cost standards as control devices results from the fact that, if standards have been fitted to operating conditions, executives are less likely to be held responsible for cost variation which is beyond their control, and are more likely to be charged with responsibility for results which are within their control.² Standards of this type can be developed only after a systematic study has been made of the actual effects upon cost behavior of various operating conditions, such as rate of output, size of manufacturing order, rate of labor turnover, etc.

The second use to which the knowledge of the behavior of per-unit cost can be put is in the reduction of cost itself. Systematic analysis of the effects upon cost of various changes in operating conditions is likely to extend the executive's understanding of the causes of cost behavior and bring to light possibilities of reduction of costs.

The third use which business men can derive from this knowledge of per-unit cost behavior is in forecasting costs and profits under anticipated operating conditions. Since profit represents the difference between total cost and total income, any improvement in the precision with which costs can be estimated is likely to result in a corresponding increase in the accuracy with which the profit-and-loss statement can be projected into the future.

Even more important use may be made by business firms of knowledge of marginal cost behavior. Most short-period managerial decisions should, if maximization of profit were the sole test of correctness, be made by balancing the additional cost caused by the suggested policy against the consequent additional revenue. As a general rule, this applies to any type of decision—the introduction of a new product, a new manufacturing method, or a new sales campaign. But most frequently utilization of cost data in this manner is essential if opti-

² This advantage holds *in toto* only when all the conditions which were allowed for in adjusting the standard fall outside the executive's realm of control and when the cost adjustment to these altered conditions is likewise not subject to his control. But important practical advantages result even when this ideal situation is not realized.

num decisions are to be made with respect to the price and the output policy of the firm and, more specifically still, when the acceptance of an additional order is being considered. The increment in revenue which the order brings should be balanced against the addition to total cost which it causes. This additional revenue produced by a unit increase in output is called "marginal revenue," just as the increase in total cost caused by a unit increase in output is called "marginal cost." Thus it appears that a knowledge of marginal cost (and of marginal revenue) is of great importance in forming the rational basis for many of a firm's most important short-period decisions.³

More and more, of late, the attention of business men has been directed to the importance of these marginal costs. Notwithstanding the fact that the behavior of cost, as output is altered, has been the object of considerable speculation, the problem, until recently, has elicited comparatively little statistical investigation. Nevertheless, it is becoming increasingly recognized in some quarters that cost compiled by routine traditional accounting procedures (with their arbitrary appraisals, allocations, and imputations) has little value as a guide to short-run business decisions and therefore has small significance in determining price.⁴

³ The significance of marginal cost and marginal revenue for short-period output adjustments has been pointed out by many economists. A precise exposition is found in Joan Robinson, *The Economics of Imperfect Competition* (London, 1933). A briefer discussion is found in Jacob Viner, "Cost," *Encyclopaedia of Social Sciences*. The long-period as well as the short-period marginal behavior is discussed by R. F. Harrod in "Doctrines of Imperfect Competition," *Quarterly Journal of Economics*, May, 1934, pp. 442-70.

⁴ It is the assumption of orthodox accounting that costs, once they have been reduced to dollars and cents, are homogeneous in behavior and co-ordinate in importance. Accordingly, a figure representing imputed rent upon an owned machine (i.e., overhead) is often considered as having the same managerial significance as a cash outlay for direct materials. Some departures from orthodox average cost accounting have, nevertheless, found their way into business practice in the form of compromises between average cost and outright marginal cost accounting. Examples of this groping for marginal estimates are seen in the concept of idle burden and in the use of standard burden rates. Both these practices bring cost estimates nearer to marginal figures when the plant is operating at less than the normal rate. Some recognition of the usefulness of such departures from orthodoxy is also being found in the business journals; cf. M. R. Benedict, "Opportunity Cost Basis of the Substitution Method of Farm Management," *Journal of Farm Economics*, XIV, 548.

It is important for management to know the short-period behavior of its average and marginal costs, yet such information does not usually exist. Orthodox accounting costs tell little about the effect of altered operating conditions upon average costs and nothing about the behavior of short-run marginal cost. Furthermore, there seem to have been surprisingly few efforts to study cost behavior systematically. A survey of published studies indicates a paucity of methodical procedures for explaining cost behavior, a widespread reliance upon rule-of-thumb guesses, and, consequently, a generally recognized inexactness of cost forecasts.⁵ Information required for balancing the expected additional revenue to be derived from a given course of action against the expected additional cost is not supplied to the management of business firms through the regular channels of accounting routine. It is derived primarily from guesses or estimates made by the enterpriser himself, or by engineers, or, more rarely still, by accountants or statisticians. Such a state of affairs invites investigation aimed at developing practical methods for studying average and marginal cost behavior, which should systematically provide management with the estimates of short-run average and incremental costs required for intelligent business decisions.

Short-period cost behavior, rather than long-period cost behavior, was made the subject of this investigation because the majority of executive decisions are probably made for the short period. Moreover, accounting costs are often moderately satisfactory approximations to long-period marginal costs.⁶

RESULTS

METHODS DEVELOPED

The results of this investigation may be summarized under two general headings: (1) methods and (2) findings concerning cost behavior. As has been previously indicated, the first objective of this study was

⁵ See chap. ii, "Survey of Previous Studies of Cost Behavior."

⁶ Long-period cost behavior was also analyzed by the author. By using procedures similar to those developed in the short-run study, long-run average and marginal cost curves both for combined cost and for several components were obtained from cost data of a chain finance company. These findings are summarized in Appendix E of the author's dissertation, "A Statistical Examination of the Behavior of Average and Marginal Costs."

to develop practical systematic methods for determining the short-period behavior of average and marginal cost. With this in mind, an examination of published studies was made in order to evaluate procedures previously employed. Few studies were found which had a direct bearing upon the problem at hand. Other studies, however, with different objectives, suggested techniques which proved useful. Procedures thus suggested were empirically tested by making a statistical study of the cost behavior of a certain furniture factory.

The particular plant chosen provided suitable data for studying the relationship between short-period changes in output (and other factors) and short-run changes in cost. Its cost data were reported at two-week intervals, and were subject to large fluctuations. Moreover, costs were available in terms of total expenditures associated with a particular total output, rather than being assigned, by means of a specific order cost system, to a particular unit of product. This total cost, furthermore, was broken down into its components (i.e., components of combined cost), each of which was recorded as a lump sum for the reporting period.

The general method which was developed as a result of the analysis of the cost data of this factory constitutes, on the whole, the most useful result of this study. In developing this method a common denominator for output was first found which would measure the great variety of models and styles produced by the factory under investigation. The term used for this measure, "old warehouse value," is one which the management of this factory used, and it has been adhered to throughout this discussion. This unit of measure is the deflated standard cost of each item, computed upon the basis of a standard quality of material (red gum), standard labor and machine efficiency, and a normal level of output. Furthermore, since it is deflated to a base period, it does not reflect fluctuations in the market price of either the finished product or the input factors. It consequently represents an approximation to a weighted physical measure of output.

The second step in the development of this general method consisted in reducing the cost data to usable form. Since the dollar was selected as the unit of measurement in preference to various possible physical measures, it was necessary to correct the cost data for changes in the prices of materials and in the wages of workers. Cost data were

then adjusted for discrepancies occurring between the time at which costs were recorded and the time at which output was reported. This adjustment was made for each of the several cost components on the basis of an individually estimated time lag. After the cost data had been thus corrected, they were, in the case of most items of cost, reduced to averages per unit of output.

The third step was to measure and study the effect of several other operating factors which influence cost behavior—such as the rate of output, size of production order, etc. This was accomplished by introducing these cost influences as independent variables in a graphic multiple correlation analysis. By this procedure the effect of each of these factors upon average cost or upon total cost was recorded in the form of a curve showing estimated costs for various levels of output, for various sizes of production order, and the like. These curves were then available for utilization in computing flexible cost standards, and for forecasting and controlling cost behavior.

The last step was concerned with the analysis of marginal cost behavior. To accomplish this the net relationship between rate of output and total cost was first obtained. This was accomplished either by direct correlation analysis of the data in the form of totals or by obtaining the average cost function and converting it into a total cost relationship. Next, this total cost function was differentiated, with the purpose of ascertaining the additions to total cost which corresponded to unit increases in output. The resulting series of additions, when smoothed, were taken as the marginal cost function.

The general methodology just outlined is, of course, an oversimplification of the procedure actually developed. As a matter of fact, alternative methods for achieving each of the four steps listed above were carefully considered and evaluated in the light of their apparent appropriateness to the particular task at hand, and the more promising of these techniques were used in the analysis of some of the cost data. These tests were chiefly applied to:

1. Alternative degrees of rectification of the cost data
2. Alternative independent variables to be used in the multiple correlation analysis
3. Alternative procedures for deriving marginal cost estimates
4. Alternative breakdowns of combined cost
5. Alternative dimensions or aspects of marginal cost

For the purpose of determining the amount of rectification which cost data would ordinarily require in order to yield satisfactory results the entire procedure outlined above was repeated with cost figures representing several stages of correction. The conclusion reached on the basis of this methodological experiment was that all data would have to be fully corrected for changes in prices, output lag, and all other operating conditions which had important effects upon cost, in order to secure workable estimates of cost behavior.

Experiments to determine which types of operating conditions were important led to the conclusion that a small number of factors could account for a large proportion of cost variance but that the importance of the several operating conditions varied for different cost components. Also, the proportion of total variation in cost accounted for by all the pertinent independent factors employed in the analysis was different for each cost item. The operating conditions which were found to be the most important cost factors were:

- | | |
|---------------------------------------|---|
| 1. Output (X_2) | 4. Changes in output from previous period (X_5) |
| 2. Size of production order (X_3) | 5. Labor turnover (X_7) |
| 3. Number of new styles (X_4) | 6. Quality of material (X_8) |

Experiments to determine the best methods for deriving marginal costs were made in two directions. The first attempted to discover whether better results could be secured when cost data were kept in terms of totals than when they were reduced to averages per unit. On the basis of experimentation with cost data it was concluded that more sensible and reliable marginal cost estimates were obtained from data expressed as totals. Marginal cost curves derived from average cost data were found to have large and probably spurious fluctuations attributable to slight alterations in the curvature of the function fitted to average costs. It is preferable, therefore, to secure marginal cost from a function which is fitted to total cost residuals rather than to average cost residuals.

Testing of alternative methods of fitting and differentiating an average or a total cost series was the other direction of this experimentation. These alternatives may be classified under three heads: (1) graphic methods—those which transform visually fitted average or total cost curves into tabular series of total cost and which then smooth the first differences secured from the total cost series to obtain

the marginal cost function; (2) mathematical methods—those which fit mathematical functions to the cost data by least-squares procedures and differentiate the resulting total cost function by means of differential calculus; and (3) hybrid methods—those which combine parts of these two methods. Conclusions reached on the basis of this methodological experimentation are as follows:

1. The graphic tabular method appears preferable in deriving marginal cost from average cost data. It is economical and easy to understand. Moreover, its greater flexibility enables it to fit complex functions and transcend accidental aberrations in the data. There is danger, however, that complex curves, fitted by either graphic or least-squares methods to crude average cost data, will result in spurious and meaningless fluctuations in marginal cost curves.

2. The derivative method may have some superiority in dealing with total cost residuals, especially when simple functions are employed. An accuracy and objectivity which are not altogether spurious can be achieved by fitting a straight line to total cost residuals. A more complex function should not be employed unless both empirical and a priori evidence indicate statistically significant curvilinearity. More reliable tests of significant departures from linearity can be applied to functions fitted by least squares.⁷

3. A hybrid of these two techniques was developed which appears to enjoy some of the advantages of each of its parents. It demonstrated several treacherous imperfections, however, in actual use with data in the form of averages which make it untrustworthy unless employed with great caution. More satisfactory results were obtained when it was used with data in the form of totals.

Another phase of the general methodological experimentation consisted in studying several component elements of combined cost to determine their average, total, and marginal cost behavior. It was hoped that the experimentation with modifications of the basic technique which were required by peculiarities in the behavior of the several cost components would prove valuable and that the resulting knowledge of the behavior of cost components would be helpful in

⁷ By fitting curves to total cost data by freehand methods marginal cost estimates which are almost as accurate are obtained in much less time. No objective test of curvilinearity is provided, however, by this method.

controlling and forecasting cost in detail and, at the same time, would serve as a check upon the accuracy of the combined cost findings.

A full statement of the conclusions reached for each of these components cannot be made here. In general, the analysis indicated (at least to the author) that detailed study of minor components did not yield results commensurate with the effort expended. Modifications in technique proved unimportant. While the behavior of the average cost of components would be of considerable managerial usefulness, their isolated, individual cost behavior is difficult to ascertain with accuracy in any real situation because of the impossibility of holding constant the inputs of the remaining cost components. The practical usefulness of the imperfect approximations obtained in this study may not be very great. The use of the findings of these subsidiary analyses as a check on the results of combined cost studies was prevented by the absence of suitable data necessary for the analysis of certain of the components.

A fifth type of exploration of methods grew out of the recognition that marginal cost has many dimensions in addition to the more generally emphasized output dimension. Since total cost is a function of many variables, cost may be regarded as marginal with respect to each of these variables. Increasing the number of new styles, for example, has a marginal cost analogous to that resulting from increasing the number of units of output. Rough estimates of some of the more important of these other margins of cost were computed, more with the hope of indicating the nature and importance of this problem and of suggesting a tentative technique than with the conviction that any very reliable cost functions had been determined.

It must be borne in mind that the conclusions regarding general procedures which have been set forth above are subject to certain limitations. For one thing, the marked superiority of one particular technique over some other may have been due, in part, to peculiarities of the special problem under investigation. Thus the methods selected may not necessarily be the best procedures for use in other investigations of cost behavior. In the second place, the methodological impediments encountered in this study may have been fewer and probably were (in large part) different from those to be met with if a similar study were made of another firm. The procedures used in this

analysis cannot, therefore, be borrowed *in toto*. However, in view of the fundamental similarity of many of the methodological problems, it is hoped that the methods developed in this investigation will be found useful in subsequent studies of average and marginal cost behavior.

It should also be emphasized that no new statistical techniques were developed in this study. The procedures proposed and illustrated in this investigation are simply new combinations of well-known methods. The only originality which can be claimed is that these procedures constitute (as far as the investigator can learn) a new approach to an important and neglected problem.

COST-BEHAVIOR FINDINGS

Although the methodological results of this study are of central importance, its findings concerning actual cost behavior are also of interest, first, as illustrative of the type of result which can be secured and, second, because of the clearer understanding thereby obtained of cost behavior in the short period.

In examining the behavior of average cost per unit the relationship between this per-unit cost and several cost factors (such as rate of output, size of lot, number of new styles, and the like) was determined by graphic multiple correlation analysis. The results were expressed in the form of net regression curves which showed the effect upon average cost of changes in each of the several operating conditions. These curves themselves provide the best type of summary for the average cost findings. The various items of cost for which such a set of curves was worked out are listed below.⁸

Corrected combined cost (Fig. 2)	Inspection labor cost (Fig. 11)
Uncorrected combined cost	Yard labor cost (Fig. 12)
Direct labor cost (Fig. 9)	Power and watch labor cost
Direct materials cost	Indirect materials cost (Fig. 13)
Indirect combined cost	Power cost (Fig. 14)
Indirect labor cost (Fig. 10)	Maintenance cost (Fig. 15)

From an examination of these figures it is apparent that the effect of the several operating conditions varies both quantitatively and

⁸ Curves for most of these cost items are shown in this abstract. Curves depicting the behavior of the remaining cost items can be found in the author's dissertation, *op. cit.*, chaps. vi and viii.

qualitatively with the type of cost. In general, the effect of altering each operating factor was about what was expected. (Exceptions probably represent errors in data or method.) Hence the contribution here made consists chiefly in a quantitative statement of expected relationships. These quantitative statements are, however, subject to serious limitations. In some instances curves have probably been fitted too closely to the observed data, thus producing an illusion of accurate knowledge when, in reality, only the general tendency of behavior is significantly indicated. In general, the scatter of observations about the curves was such that a wide band of standard error must be drawn about the curve indicating that only its general level and direction are statistically reliable.⁹

The average cost findings of combined cost are presented in Figures 2 and 3, which are discussed in chapter vi. The observed average cost behavior of several components of combined cost is presented in Figures 9, 10, 11, 12, 13, 14, and 15, which are dealt with in Appendix A. These findings provide the raw material for building up flexible cost standards, for obtaining a clearer understanding of the causes for cost behavior, and for forecasting expenses more accurately. This raw material, however, must be adjusted and modified in different ways for different purposes. Problems encountered in making these adjustments are discussed in chapter viii.

Not only the average cost behavior but also that of marginal cost was determined for each of the foregoing expense items. The magnitude of marginal cost was computed at each of a series of output levels and was expressed either as an equation, a curve, or a series of marginal cost magnitudes, depending upon the technique of derivation employed. Marginal cost estimates for combined cost were obtained from data representing several levels of correction. Marginal estimates were also computed by several alternate procedures from fully corrected data in order to evaluate the various methods. These findings for combined cost are presented in Figures 4, 5, 6, 7, and 8 and in Tables IV and V. Since they are fully discussed in chapter vii, it is sufficient at this point to mention only a few of the general characteristics of the findings.

⁹ See chap. v, "Reliability of Findings," for a more detailed discussion of these limitations.

In the first place, marginal combined cost appears to be constant throughout most of the usual range of output,¹⁰ but appears to rise as the output falls toward zero. In the second place, estimates of marginal combined cost obtained from total cost data representing several levels of correction varied considerably and improved in reliability with each successive stage of data rectification. From this we can assume that complete refinement of the data was necessary in order to obtain satisfactory results for this firm during this particular period. In the third place, the curves of marginal combined cost which were obtained by the several alternative methods differed considerably in their fluctuations (we may consider these fluctuations as obviously spurious), but held to the same general level and exhibited considerable uniformity with respect to an upturn for small rates of output.

The marginal cost estimates for components of combined cost are shown in the form of equations and tables. The undulations of most of these marginal cost curves are probably not significant but are attributable, in part at least, to the use of data in the form of averages and to errors in sampling, curve-fitting, and accounting classification. These findings are presented in Tables XIII-XXII inclusive in Appendix B.

In closing, the limitations and inaccuracies of these cost findings should be emphasized. The treacherous nature of social-science data, the questionable reliability of the sample, and dubious appropriateness of many of the procedures employed in analyzing the data have often led to grave misgivings in the mind of the investigator as to the statistical significance of many of the findings of this study. But despite the fact that the estimates of average and marginal cost behavior for this furniture factory are not completely reliable, they are nevertheless considerably more trustworthy than the crude guesses which preceded them. Moreover, a method has been developed which, it is hoped, will, with repeated study of similar data, yield increasingly reliable results.

¹⁰ Marginal cost will remain constant as output increases if average cost is made up of a constant term plus a hyperbolic function—in other words, if variable costs remain constant per unit and fixed costs per unit decline; see Jacob Viner, "Cost and Supply Curves," *Zeitschrift für Nationalökonomie*, III, No. 1 (September 25, 1931), 1-48.

CHAPTER II

SURVEY OF PREVIOUS STUDIES OF COST BEHAVIOR

This summary of a survey of the available literature dealing with cost behavior is restricted to those studies which are concerned primarily with the effect of output upon cost. Output is usually one of the most important factors causing variation in total cost and in average cost¹ and is the significant factor in deriving marginal costs.² Comparatively few systematic studies of the effect of other factors upon cost were found. These are cited in connection with the net regression curves for the respective cost influences.

As opposed to the numerous studies of average and total cost, there was a surprising dearth of studies of marginal cost. This is especially noteworthy in view of the importance of marginal cost to general price theory and its even greater importance to internal management of business firms. Studies of this type have, however, in all probability been made by private firms, but have not found their way into print because of the confidential character of the data.

Two basic types of methodologies may be distinguished in these previous studies of cost behavior: (1) that which relies heavily upon rule-of-thumb estimates for arriving at the cost at various output levels; and (2) that which statistically analyzes actual cost data at various levels of output. The demarcation between these two methods is, of course, somewhat arbitrary.

¹ This statement is not incompatible with the estimate, for the firm studied, that marginal cost is constant with respect to output. A constant variable cost per unit together with a hyperbolic fixed cost function will yield constant marginal cost yet permit variability in average and total cost.

² Cost increments can also be computed for other independent factors, such as the size of lot, number of new styles, etc. But the increment associated with output is most important and most familiar and is, therefore, the one usually meant by the term "marginal cost."

METHOD OF ESTIMATES

The first methodology, the method of estimates, is based primarily upon a reclassification of cost accounts according to their degree of variability in response to changes in the level of output. Two variants of this method can be discerned: (1) a twofold classification of all cost items as either absolutely fixed or proportionately variable; and (2) a tripartite classification which includes, in addition to these two categories, an intermediary one composed of semivariable or composite items.

Although the twofold concept of constant and variable cost is convenient, it is oversimplified. By defining constant cost as that part of cost which continues when the plant is idle,³ and by assuming that all other expenses vary proportionately with output, an important intermediate type of cost has been ignored. It is true that many overhead costs do fall neatly into one class or the other. Nevertheless, there are many expenses which are intermittently variable and which fluctuate in discrete steps at certain critical resistance points.

Both J. O. McKinsey⁴ (in 1923) and John H. Williams⁵ (in 1922) early recognized the need for introducing a third, intermediate category of semivariable expense. This recognition of the complexity of cost behavior led to the development of more elaborate methods of compiling and portraying cost estimates. To this development the advocates of the break-even chart added much. The major contributions of this group are: (1) acknowledgment of the need for an intermediary category of semivariable costs; (2) more explicit definition and isolation of fixed costs; (3) recognition of the controllable character of many fixed costs; (4) stimulation, by the development of the flexible budget, of continuous improvement in the accuracy of estimates of the effect of varying volumes upon cost; and (5) development of a graphic method for portraying the relationship between costs, profits, and output.

Of all these contributions of the break-even chart advocates per-

³ C. B. Williams, "Treatment of Costs during Periods of Varying Volumes of Production," *Journal of Accounting*, XXXII (1921), 329-33.

⁴ *Budgetary Control* (New York: Ronald Press Co., 1923), p. 193.

⁵ "A Technique for the Chief Executive," *Bulletin of the Taylor Society*, VII (1922), 47-68.

haps the most significant is the clear, vivid presentation of the way in which costs, sales, and profits vary with output. In a break-even chart the abscissa records rate of output, and the ordinate, the dollars of total sales revenue or total cost. Both sales revenue and total cost are assumed to rise at a constant rate and are hence drawn as upward sloping straight lines which intersect at a break-even point, beyond which lie profits.⁶

Certain limitations of this break-even chart technique should, however, be pointed out. Its basic weakness lies in the methods used to arrive at the cost estimates which it so ably portrays and manipulates. Unrealistic assumptions of a sharp dichotomy of perfectly fixed and proportionately variable components, of the complete independence of cost components from one another, and of a monistic, functional relationship between cost and output are usually tacitly and unquestioningly made. The assumption of a cost component which is proportionately variable at a constant rate, regardless of the time span of output variation, sets aside the principle of diminishing returns and ignores the effect of varying lengths of time required for the adjustment of different cost items to altered output. Moreover, cost is a function of many variables, of which output is but one. By ignoring these other factors this method tacitly assumes that they do not exist or, at least, that they do not significantly affect the output relationship.

Little effort appears to have been expended in systematic analyses to improve the accuracy of estimates of cost behavior, and little recognition appears to have been given to the limited accuracy of estimates made without the benefits of such analyses. These weaknesses in the methods of estimating cost behavior have been corrected in some measure by important refinements in the procedures of analyzing the hitherto unstudied semivariable items. It is these composite items which give trouble since a large proportion of the cost items fall in this intermediate class and since the perfectly fixed and proportionately variable costs are not difficult to forecast. In general, an attempt is

⁶ The essential principle is thus stated by C. E. Knoeppel, "Must Fixed Expense Be Fixed?" *Factory and Industrial Management*, September, 1931, p. 323: "While the sales income line begins at zero and crosses the 100% capacity line at a given point, the total cost line does not begin at zero, but at a point where fixed costs end. Somewhere up the capacity scale costs and sales lines cross; profits lie above this cross-over point; losses below."

made to divide the composite item into a perfectly fixed portion and a proportionately variable addition. Five different methods of accomplishing this separation may be mentioned:

1. By means of rule-of-thumb estimates
2. By means of estimated cost amounts at maximum and minimum output levels
3. By means of the actual cost amounts at typical high and low output levels
4. By means of a series of separate estimates of the amount of each individual composite item at each of a series of production levels
5. By means of an analysis of the time span of variability of each composite item

Improvement in accuracy of rule-of-thumb estimates as a result of experience in budgeting was noted by Joseph Geschelin.⁷ G. E. F. Smith⁸ and J. H. Williams⁹ sponsor the method of estimating cost amounts at maximum and minimum output levels, and then interpolating between these extremes. A third method of breaking composite items into fixed and variable portions is advocated by H. P. Dutton. He employs the same cost formula in his analysis that Williams used, but, instead of utilizing estimated costs, he takes the actually recorded cost figures of the composite item for two operating periods which differ materially in rate of output.¹⁰

Still another method of segregating the variable components of a composite cost item could be developed by the use of hyperbolic cost paper. R. M. Barnes describes the advantages of this ingeniously ruled paper in an article which, however, does not mention this particular use.¹¹ The scale of this paper is so designed that the rectangular hyperbola of average fixed cost per unit assumes the form of a straight

⁷ "Three Budget Compasses Give S K F Industry Smooth Sailing," *Automotive Industries*, April 30, 1932, p. 211.

⁸ "Budgeting Simplified by Separating Fixed and Fluctuating Costs," *American Accountant*, XVI (1931), 40-45.

⁹ *Op. cit.*, pp. 47-68. He suggests here a formula for interpolating such a cost in the intermediate range: Cost = (Expense at minimum expected level of output) + (Proportional cost as a percentage of sales volume).

¹⁰ "Accounting for Fixed Expenditure," *Factory and Industrial Management*, LXXVI (1928), 914-16.

¹¹ "Smoothing the Road to Predetermined Costs," *ibid.*, LXXIX (1930), 1069-70.

line sloping downward to the left. The per-unit cost curve of a directly variable cost appears as a horizontal line. Floyd F. Hovey makes the suggestion that the per-unit cost curve of a semivariable cost will be a sloping straight line whose ordinate at infinity will show the per-unit value of its variable portion.¹²

A system of handling composite items which is very largely free from the questionable assumption that the fluctuating residue of a composite cost is proportionately variable over infinitesimal steps (an assumption implicit in the technique just noted and also in the Dutton method) has been suggested by J. M. Clark,¹³ Walter Rautenstrauch,¹⁴ G. H. Bates,¹⁵ and others. Clark, for example, suggests that an estimate be made of each subdivision of cost for each of a series of outputs on the basis of a survey of actual cost experience, and also on the basis of an appraisal of current conditions. The total cost series thus built up makes no a priori assumptions regarding the nature of cost behavior. Since the result is frankly a forecast of how costs will behave, rather than a chronicle of how they actually did vary, it is perhaps free from some of the defects of a mechanical extrapolation of past experience. Such an estimate, of course, like any other, can attain only approximate accuracy.

G. H. Bates shows a clear comprehension of the value of understanding how cost varies with output. He has made some distinct contributions to the methods of analysis by constructing a variable budget for the Staten Island Shipbuilding Company, which was designed primarily to give the following information: "a standard of expense to cover any volume of business, . . . a determination of overhead in percentages for differing volumes of business, . . . [and] a disclosure of additional expenses occasioned by increasing the volume of business."¹⁶ The contributions made to the methods of estimating cost variation

¹² "Cost Accounting and Budget Making," *Bulletin of the Taylor Society*, XVI (1931), 102.

¹³ *The Economics of Overhead Cost* (Chicago: University of Chicago Press, 1923), p. 180.

¹⁴ "The Economic Characteristics of Manufacturing Industries," *Mechanical Engineering*, November, 1932, pp. 830-55.

¹⁵ "Actual Examples of the Advantageous Use of Operating Budgets," *Yearbook of the National Association of Cost Accountants* (1923), pp. 140-44.

¹⁶ *Ibid.*, p. 140.

consist of: (1) a detailed schedule of estimated expenditures at frequent output intervals; (2) a variable burden rate for each department; (3) an allocation of fluctuating, service department expenses to the direct departments on a variable basis; and (4) a clear recognition of the principle of marginal costs.

A few studies employing the method of estimates have been carried out with the clear objective of determining marginal costs. J. M. Clark's schedule of estimated cost at a series of output levels was designed specifically to illustrate a method of computing these incremental costs. The difference between the total cost at one level and that at the next higher level was taken as the increase in expense, or the differential cost.¹⁷ G. E. F. Smith advocates the use of his flexible budget for somewhat similar purposes. A sales order, the price of which covered the variable direct costs plus the fluctuating overhead, could be profitably taken despite the failure of its price to cover average cost. That is to say, per-unit variable cost may be used as a rough equivalent of marginal cost.¹⁸ G. H. Bates also saw the application of his flexible budget to the problem of deciding upon the acceptance of low-priced business. To facilitate this application, Bates used his budget figures to work out a fairly accurate schedule of estimated differential costs for particular departments at various levels of output.¹⁹ T. H. Saunders likewise suggested the construction of an elastic budget as a means of arriving at an estimate of marginal costs.²⁰

The pertinence of differential (or marginal) cost in deciding the question of the acceptance of additional business at a given price has been recognized by some engineers. Estimates of this cost increase have been developed, partly on the basis of known physical input and output relationships consequent upon machine production and partly on the basis of judgment and experience. The accuracy of many of these estimates of marginal cost is questionable. In all probability they have frequently been too low because of the tendency to treat immediate out-of-pocket outlays as though they represented the entire incremental costs. This neglect of variable and postponable elements

¹⁷ Clark, *op. cit.*, p. 185.

¹⁸ Smith, *op. cit.*, pp. 40-43.

¹⁹ Bates, *op. cit.*, pp. 143-44.

²⁰ "Overhead in Economics and Accounting," *Bulletin of National Association of Cost Accountants*, VII (1926), 587.

of overhead cost has often resulted in an underestimation of marginal costs.²¹

METHOD OF STATISTICAL ANALYSIS

A systematic statistical analysis of the actual cost data for past periods is one very fruitful way of developing standards for estimating future cost behavior. Hence it is surprising to note the paucity of studies employing this statistical approach. Those few statistical studies which were uncovered fall under two general heads: (1) those primarily concerned with average or total cost behavior and (2) those whose principal objective is the examination of marginal cost behavior. Each of these two general headings can be still further subdivided into: (a) studies which recognize only simple relationships and (b) studies which make specific use of multiple relationships.

Data used in these four kinds of studies are of two types: (1) simultaneous observations of a number of similar plants (i.e., all observations were taken at exactly the same time) and (2) successive observations of the same firm spread out over a period of time. These observations may produce either long- or short-run results, depending upon whether it is the scale of the plant or the intensity of its use which has changed between successive observations. Difficulty is usually encountered in isolating variation which is purely short-period or purely long-period.

STUDIES OF AVERAGE AND TOTAL COST

An example of simple relationships determined from data of the second type (i.e., successive observations of the costs of the same firm) is seen in an analysis made by D. D. Kennedy of the expenses of a machine shop over a period of years. By computing the coefficient of correlation between each expense item and direct labor hours (a supposedly variable item of cost) a classification was developed of the individual items of overhead according to their variability with production.²² The classification thus built up could have been used (though this application was not contemplated by Kennedy) to formulate rough estimates of average and marginal cost at a series of output

²¹ E. L. Grant, "Increment Costs and Sunk Costs," *Mechanical Engineering*, October, 1934, p. 586.

²² "Variability of Overhead Costs," *Journal of Accounting*, XLIX (1930), 202-6.

levels. The procedure followed by Kennedy is inadequate for our purposes in that the regression is simple rather than partial and is not related to output but to an item of variable input. It is consequently of limited usefulness in determining the pure relationship between average or total cost and the rate of output. Thus it is really of little direct value in computing marginal cost unless it can be assumed that direct labor hours vary proportionately with, and therefore accurately measure, output.

Instead of leaning heavily on estimates of the supposed behavior of overhead cost D. S. Cole used actual overhead at a series of output levels in developing a linear relationship. After skimming off fixed overhead items, he plotted the average monthly totals of variable overhead against the average monthly productive labor hours (his measure of the rate of production). He then drew a freehand line which represented the average relationship between overhead cost and a supposedly proportionately variable input factor.²³ The limitations of this simple but crude method are obvious. The curve was roughly drawn upon the basis of a few yearly averages which concealed within the year much significant variability in cost. Moreover, it represented a gross rather than a net relationship since no allowance was made for the influence of cost factors other than the rate of production.

The foregoing statistical studies treated cost behavior as though it were a simple function of output. A more realistic view of cost fluctuations can be taken if the effect upon cost of influences other than output is clearly recognized.²⁴ Such factors as size of the production order, level of wages and prices, efficiency of the labor force, and number of new styles manufactured are all constantly affecting cost variation and thereby distorting the pure relationship between cost and output. A complete grasp of the changing cost situation can be obtained only through an understanding of the influence of changes in these variables upon the resulting cost.

Two general methods are available for allowing for the distorting effect of these other influences. One is to remove the effect of these

²³ "Taking the Guess out of Overhead Costs," *Industrial Management*, LXV (1923), 324-26.

²⁴ Floyd F. Hovey, e.g., in his "Present Cost Facts Graphically" (*Factory and Industrial Management*, LXXXI [1931], 599-601) recognizes the complexity of the causes for cost fluctuation.

distorting factors upon the cost data before attempting to analyze the relation of cost to the rate of production. A second method is to measure such distorting influences at the same time that the analysis of the major relationship of cost and output is made. This can be done by means of multiple classification, multiple correlation, or similar techniques. In actual practice, however, these two methods can best be combined. Data which have been corrected for certain dynamic variables, such as price changes and long-term trends, are then subjected to a multiple correlation analysis in order to stabilize the effect of still other factors not conveniently removed from the data.

STUDIES OF MARGINAL COST

In addition to these analyses of average and total cost some studies of marginal cost were found. Even though not directly focused upon the determination of incremental costs, quantitative farm management studies have, nevertheless, made several significant contributions to the problem under investigation. These contributions may be classified under three general heads:

1. The techniques of multiple correlation, joint correlation, and multiple classification have been refined and developed through much use. (This is, perhaps, the most important contribution of all the farm cost studies.)

2. Increments in physical products associated with physical increments in input factors have been computed in an effort to aid the farmer in balancing marginal product with marginal input.

3. Regressions and part correlations have been established for the relationships between labor income and size of farm (or of component enterprise). The direct usefulness of these regression lines in determining marginal cost is relatively limited, however, because of the heterogeneity of the cost data, the difficulty of measuring size, and the irrelevance of the dependent variable commonly used, namely, labor income.

In addition to these agricultural studies a few industrial analyses were found which determined marginal cost by statistical methods. The most outstanding of these contributions which came to the attention of the investigator were made by W. L. Crum, R. T. Livingston, and T. O. Yntema. By determining multiple regression equations for the effect of passenger and freight traffic upon the operating costs of

railroads Crum secured a measurement of the increment in cost which was occasioned by a ton-mile of freight traffic.²⁵ This marginal cost was constant with respect to output since only linear regressions were determined. But the irregularity of the data made this first approximation appear sufficient. Only two independent factors (both of them phases of output) were employed in the analysis on the ground that other influences were probably randomly distributed with respect to the relationship under investigation.

R. T. Livingston fitted a straight line, by the method of least squares, to data representing the total operating expenses of a steam-generating plant at various levels of output. As an indication of the degree of success attained by this simple linear correlation he computed the standard error of estimate and the coefficient of correlation. As one aspect of the economic significance of this linear equation of total cost he computed marginal cost, which he found to be constant for all levels of output, since the total cost regression was linear.²⁶

This remarkable statistical determination of marginal cost employed a simple and objective technique which doubtless will yield useful practical results under certain conditions. There are, however, certain limitations to its general applicability. First, no allowance was made for the effect of changes in prices and in wage rates upon the historical total costs used as data.²⁷ Second, no provision was made for allowing for the time which often elapses between the recording of a cost and the recording of output. Third, no effort was made to hold constant various other distorting factors, by means of multiple correlation or multiple tabulation. And, last, no attempt was made to investigate the realism of the assumption that the total cost function is a straight line. This assumption of constant marginal cost appeared to be reasonable from Livingston's data; however, the output range was limited. The a priori expectancy was that the operation of the law of diminishing returns would cause the marginal cost to rise as the plant

²⁵ This marginal cost interpretation, however, was not given to the results by Professor Crum; see his "Statistical Allocation of Joint Costs of Railroad Freight and Passenger Service," *Journal of the American Statistical Association*, XXI (1926), 18.

²⁶ "Control of Operating Expenses," *Mechanical Engineering*, LIV (1932), 711-17.

²⁷ The prices of input factors frequently constitute an important distorting influence upon cost behavior. Effective wage rates are sometimes positively correlated with output, however, and hence are one factor which would cause marginal cost to rise (see chap. iii, n. 9).

was pushed to capacity. The linearity of the total cost function should therefore be tested, not merely assumed. And even if it is established, the limited output range over which it is likely to hold should be specifically recognized.

Some of these limitations of the simple linear correlation method were not found in a multiple curvilinear correlation procedure used by T. O. Yntema²⁸ and W. W. Peterson.²⁹ Allowance for the distorting effect of various cost influences other than output was made by means of graphic curvilinear multiple correlation analysis. The flexible net regression curve of average cost upon output, which was secured by this procedure, made no presuppositions regarding the shape of the marginal cost curve. This net regression curve for average cost was then converted into a total cost function. The first differences of this total cost function were then computed and smoothed to secure a marginal cost curve.

The technique developed by Yntema for investigating long-period cost behavior was made the basis for the methods used in this study of short-period costs. However, since the problems are, in many respects, different, the procedure was amplified and modified in several respects. First, the cost data were laboriously rectified before commencing the multiple correlation analysis. This was found to be necessary because the data used in the present investigation appeared to be markedly affected by variations in prices and wages, and by lag in recording output. (Errors of this type were less serious in the data used by Yntema since short-period fluctuations tended to wash out in the annual averages which he used as data.) Second, several alternative procedures were tested and compared. Alternative degrees of rectification of the cost data, alternative methods of curve fitting, and alternative procedures for deriving marginal cost were tried out in order to note the differences produced in the results. Third, in addition to the analysis of the average and marginal behavior of combined cost, several of the more important components of combined cost were also analyzed. And, fourth, other margins of cost, in addition to the traditional output margin, were computed.

²⁸ Unpublished computations of marginal cost for a finance company.

²⁹ "A Study of Factors Influencing the Cost of the Small Loan Business," a Master's thesis written under the direction of Theodore O. Yntema, University of Chicago, 1934.

CHAPTER III

METHODS OF PREPARING COST DATA FOR ANALYSIS

An examination of previous investigations of cost behavior indicated that these studies failed to emphasize two of the most difficult problems in short-period cost analyses, namely, (1) the selection of the cost data and (2) the correction of defects in these data. The methods followed in this study in the collecting and sifting of the cost data, and then in correcting these data for the effects of irrelevant influences, are discussed under the following headings:

1. Collection of cost data
 - A. Selection of establishment whose costs were studied
 - B. Selection of cost items for which data should be collected
 - C. Selection of unit for measuring output
 - D. Selection of unit for measuring cost
2. Correction of defects in cost data
 - A. Distortion prevented by holding certain disturbing factors constant
 - B. Distortion caused by some of the irrelevant variables removed from the cost data
 - C. Distortion caused by certain factors allowed for by introducing them as independent variables in a multiple correlation analysis

COLLECTION OF COST DATA

SELECTION OF ESTABLISHMENT WHOSE COSTS WERE STUDIED

In studying the short-period cost behavior of a particular business firm it was essential that an establishment be found which met with all the requirements for such a study. A factory in a small city, manufacturing medium-grade furniture, was decided upon. It made available to the investigator complete, detailed records, covering several years, of the actual costs for each two-week accounting period. Moreover, these years included frequent and drastic changes in the level of output and in the magnitude of the several cost items, without any accompanying changes in the scale of the plant.

Of the several plants operated by this company two were selected

for investigation. They presented more difficult methodological problems than did the others because they were the largest, manufactured a greater variety of products, and experienced wider fluctuations in output and in size of production order. These two plants were treated as one unit because they were located side by side; had products which were almost identical; were substantially alike in degree of mechanization and in methods of organization; were fed by the same veneer mill and dimension mill; and, lastly, because they were really operated together (since one method of running part time was to shut down one plant completely).

SELECTION OF COST ITEMS FOR WHICH DATA SHOULD BE COLLECTED

Only factory cost data were collected for analysis since information concerning selling and general administrative expenses was not made available to this investigator. These data were collected in as detailed an expense breakdown as the records would permit.¹ This was done because separate correction of the individual components seemed to improve the accuracy of these adjustments, and because separate correction was a necessary preliminary to a separate study of the cost behavior of these components, which study promised to broaden the methodological experimentation and enhance the usefulness of the entire study.²

SELECTION OF UNIT FOR MEASURING OUTPUT

The problem of selecting the proper unit for measuring output is an important one because output is the factor whose relation to cost is of primary concern in this study. This problem is a difficult one because of the great variety of products which constitute output and the numerous dimensions according to which output could be measured.³ No single one of the many dimensions of output will be precisely suitable for all elements of cost. However, since the determination of the combined marginal cost of increases in the quantity of output is

¹ Previous statistical studies of cost behavior have not, so far as the author can learn, carried the analysis to the constituents of combined cost. See chap. ii for a discussion of these studies.

² See chap. iv of the author's thesis, "A Statistical Examination of the Behavior of Average and Marginal Costs," for an elaboration of this statement.

³ See J. M. Clark, *The Economics of Overhead Cost*, p. 50.

one of the principal objectives of this study, it is necessary to discover some one measure of output which will serve as a common denominator for the various types of finished product.

Output may be reflected by some input item which is proportional to output⁴ or it may be measured more directly by some index of output. The various alternative measures which were systematically studied may be divided into two main types of units—physical units and value units. These may be further subdivided as follows:

Physical Units	Value Units
Measures of input	Measures of input
Board feet of lumber	Value of direct materials
Direct labor hours	Dollars of direct wages
Measures of output	Measures of output
Number of board feet	Wholesale price
Number of cases (pieces)	Standard cost
Number of each type of case	Old warehouse value

Previous studies of cost behavior throw some light upon this problem of choosing among these various measures of production activity. In general, it may be said that in agricultural studies physical units prevail. They are also used quite commonly in industrial studies as measures of input and, occasionally, as measures of output for firms free from product diversity. Value units, however, are used more frequently in industrial studies, particularly as measures of output.

After examining each of the foregoing alternative units for the purpose of determining a specific measure of production activity, no one of the physical input units listed for the furniture factory was considered quite suitable. Board feet of lumber was unsatisfactory because it is not a dominant input item and does not accurately reflect the relative cost-causing importance of the various articles produced. Direct labor hours were also rejected, partly because of the disparity of wage rates among departments, partly because of the relatively small importance of this input factor, and partly because it was not proportional to output. In a like manner physical measures of output were also found wanting. Number of board feet of output had the same

⁴ Input measures are strictly suitable for determining marginal costs only when the input item is proportional to output. They have, nevertheless, been employed as a rough index of the level of output when the variety of finished product is so great as to prevent a common denominator.

limitations as board feet of input. Number of cases was insensitive to variation in the type of piece produced. An attempt to correct this defect by classifying cases by product types and thereby developing a weighted physical unit was unsuccessful because of the great variation in value which was found to exist within the type.

Value units appeared to be more promising. Value of direct materials is a more inclusive measure than a physical measure of input, and, what is more, it reflects variations in the complexity of product more accurately. However, this measure also reflects changes in the price of materials—a type of fluctuation which is irrelevant to the relationship under consideration but which could be removed by deflation. Perhaps the most satisfactory value measure of the input type is dollars of direct wages. However, it too is affected by rate fluctuations and fails to reflect output accurately.

Among the value measures of the output type, wholesale price and standard costs appear to have serious limitations. Wholesale price data are exceedingly laborious to collect and free from the numerous trade and credit discounts. Moreover, these data reflect shifts in the demand schedules for the various finished products and in the purchasing power of money. Standard cost, on the other hand, is free from some of these defects (e.g., it does not reflect shifts in the demand for finished products). But then, again, it is sensitive to changes in the prices of various input factors (since it is revised frequently to account for such changes). Consequently, standard cost has many of the deficiencies of a market-value measure of output.

Old warehouse value, as a value measure of output, is definitely superior to the other two value units just mentioned. It is a deflated standard cost computed for each separate model of furniture by summing the three following normal cost components:

1. Standard labor cost, deflated to the 1925-base wage level
2. Standard material cost, figured on the basis of standard quality red gum and similarly deflated for price change
3. Machine overhead, allocated upon the basis of a normal level of output

It is important to notice that, once a warehouse value has been established for any given item, it remains unaltered regardless of changes in the rate of output, size of lot, and the like, and is changed only by major alterations in production procedures. This output unit is also

unaffected by changes in the prices either of raw materials or of finished products.

Thus we can see that old warehouse value is superior to standard costs in that it is wholly unaffected by changes in the wages of labor, in the prices of materials, and in the quality of materials. And, unlike wholesale prices, it is impervious to shifts in the demand schedules of finished products, to changes in the purchasing power of the dollar, and to alterations in the discounts deducted consequent upon shifts in the channels of distribution. From one point of view total old warehouse value is an index of physical inputs under constant price and operating conditions. From the point of view pertinent to this study, however, total old warehouse value is a weighted sum of the various items of output and, consequently, is an index of physical volume of output.

SELECTION OF UNIT FOR MEASURING COST

Selection of a unit for measuring cost was not difficult. The choice lay between physical and monetary units. Physical units had certain significant advantages: (1) no allowance would need to be made for changes in price level; (2) utilization of these results in making business decisions regarding future periods would be facilitated because the difficulty engendered by both absolute and relative price shifts would be obviated; (3) permanence of results would be increased because relatively changeless physical units would be employed rather than any unstable monetary unit.

Difficulties, however, barred the way to the use of physical units in this case. Complete cost data were not available in physical terms, and the difficulty of combining the cost elements thus measured was unsurmountable owing to the fact that there were thousands of types and qualities of cost constituents. Obviously, each of these constituents could not have been separately analyzed in terms of its particular physical unit. Hence, these diverse physical units would have had to be combined in proportion to their relative values at some base period. The resulting physical unit would have little superiority over the monetary unit in coping with absolute and relative price changes.

It was therefore decided to use the deflated dollar as the common denominator of cost. The dollar was the unit in which the data were most completely available, and also the unit which was familiar to executives and truly significant for business decisions.

CORRECTION OF DEFECTS IN COST DATA

After the cost data had been selected and the units of measurement chosen, the most obvious defects in the data were removed before the systematic analysis was begun. The methods used to free the cost-output relationship from the distorting effect of irrelevant variables may be grouped under three main heads:

- A. Distortion was prevented by holding certain disturbing factors constant
 - 1. Factors which would have obscured the cost-output relationship were held constant by careful selection of the period for which the data were studied
 - 2. Cost items which could not be corrected for the serious effects of irrelevant influences were omitted
 - B. Distortion caused by some of the irrelevant variables was removed from the cost data
 - 1. Effects of certain disturbing variables were "averaged out"
 - 2. Certain cost items were stabilized at their three-year average
 - 3. Many cost elements were adjusted by means of index numbers of prices and wage rates
 - 4. Portions of certain costs were reallocated to the accounting period in which the output they contributed to was recorded
 - C. Distortion caused by certain factors was allowed for by introducing these factors as independent variables in a multiple correlation analysis
- The first two of these groups of methods will be discussed in this chapter, and the third, which may more properly be regarded as a method of analyzing cost behavior, in the next chapter.

DISTORTION PREVENTED BY HOLDING CERTAIN DISTURBING
FACTORS CONSTANT

By judicious selection of the period for analysis certain disturbing variables, whose effects are very difficult to remove, can be reduced approximately to constants. Three years were chosen during which technological progress in these plants was almost at a standstill and in which no growth of "fixed plant" occurred. Continuance of the same group of mature executives, moreover, kept managerial skill at a fairly constant level. Finally, accounting routines were not altered in any fundamental respect. What changes did occur in overhead allocation were allowed for.

Certain items of overhead cost were dropped from the analysis because their fluctuations were relatively great as well as irrelevant,

and also were difficult to correct by other means. Most of these items depended upon arbitrary decisions of the accounting department and hence fluctuated erratically and irrelevantly. Items handled in this way were: (1) prorations of plant superintendence cost to other minor plants; (2) central administrative cost prorated to plants studied; (3) teaming and trucking among plants; and (4) traveling expenses of buyers and superintendents. Certain accounting periods were also dropped from the sample in order to remove the effect of a particular disturbing variable. Such action is dangerous, and is only justifiable when the distortion thus removed more than compensates for the loss in size and representativeness of the sample.

DISTORTION CAUSED BY SOME OF THE IRRELEVANT VARIABLES
REMOVED FROM THE COST DATA

The two procedures discussed above attempted to prevent distortion of costs by holding constant the disturbing variables themselves. The second group of methods attempted to remove distortions from costs by means of direct modification of the data. This procedure was employed when the consequences of various disturbing variables could not be prevented and when the approximate effect of these irrelevant factors upon the cost data could be rather definitely determined.

A satisfactory sample being given, the effects of certain disturbing factors tend to "average out." This holds true, however, only when the errors are small and evenly distributed with respect to the relationship under investigation.⁵ If the errors caused by disturbing influences affect only the values of the dependent variable and are not correlated with its true values, their presence does not tend to change the slope of the regression lines from their true slope. It does, however, tend to lower the correlation and increase the standard error of estimate.

A cost item was stabilized at its three-year average when it became clear that the causes for its fluctuation were not related to output. This was the case for three items of overhead cost: (1) depreciation,

⁵ In a study of joint costs in railroad operation W. L. Crum used only two independent variables and made no modifications of data. This was defended on the ground that disturbing variables would tend to be distributed sufficiently evenly so as to have negligible effects ("Statistical Allocation of Joint Costs of Railroad Freight and Passenger Service," *Journal of the American Statistical Association*, XXI [1926], p. 22).

(2) property taxes, and (3) fire insurance. The variation in depreciation rates was not due to differences in intensity of use but rather to the firm's practice of ceasing to charge depreciation when a machine was fully depreciated. The cost-output relationship was in no way affected when a machine's useful life exceeded its estimated life. Variation in property taxes was a function of the judgment of appraisers and of the city tax policy. It was not a function of output. Fire-insurance rates moved up and down with the success of the town's fire department, in a manner wholly unrelated to changes in production activity.

Since there were violent fluctuations in material prices and in wage rates during the period chosen for analysis, it was necessary to correct the cost data for changes in the prices of these input factors. Construction of an index number for direct materials⁶ involved problems of collecting data, sampling, weighting, and averaging. In collecting the data price quotations were secured from the purchase records. Whenever there were purchases at more than one price during any given two-week accounting period, a weighted average of these prices was taken.

⁶ It should be emphasized that the attempt to remove from the data the effect of all price fluctuations is based upon the assumption that these price fluctuations are not a function of the output of the particular firm. The indications are that this assumption is not entirely correct. It appears reasonable to suppose that there is some positive correlation between the rate of output and the effective prices paid for the input factors (labor and material). If the output rate of this concern is positively correlated with the output of the furniture industry and if the output of the industry is positively correlated with the prices of input factors, then the output of the concern is related to the prices of cost components even though competition were approximately atomistic. If the market is monopolistic, that is, if this firm represents an appreciable proportion of the demand for a certain type of factor, then its price may fluctuate more directly and markedly in response to changes in the output of this firm. For example, executives are of the opinion that savings from quantity discounts on certain materials are more than counterbalanced by premiums demanded for rush delivery required when output is great. Similarly the effective price paid for certain types of labor may fluctuate roughly with output, because this firm's requirements constitute the most important demand for this labor in the town in which the company is located. Even though wage rates are not very flexible, the quality of worker obtained at a given rate does appear to vary with output; this results in variation in the price per unit of work. To remove these price changes which are caused by output fluctuations results in an erroneous estimate of the effect of output upon cost.

A completely accurate index of material prices would include every one of the hundreds of items of material used in the manufacture of furniture. Since this was out of the question, a representative sample was selected. Commodities for which price-making forces were similar were grouped together, and samples were selected to represent each of these groups. The criteria used in selecting these group representatives were: (1) continuity of usage, (2) constancy of the percentage of total material costs, (3) typicalness of price fluctuations, (4) relative importance, and (5) frequency of price quotations.

The prices of the samples which had been thus selected as representatives of groups were then converted into relatives with January, 1932, as the base. In combining these relatives a weighted arithmetic mean was used and the price relatives of sample commodities were weighted according to the 1932 value consumption of the commodity group which they represented. This average was considered sufficiently accurate for the purpose of this investigation.

Construction of index numbers for indirect materials involved problems similar to those dealt with in computing an index number for direct materials. But two differences in the methods of computation are significant. In the first place, no single index number was developed to represent all the indirect materials; and, in the second place, some of the indirect material price indexes were much cruder than the direct material price index.

The index numbers thus constructed possess several limitations. In the first place, they do not correct for variations in price alone, but tend also to adjust partially for some of the differences in the quality of the materials. Furthermore, weights which reflect the relative importance of usage of a commodity in a base period may not measure usage at some later period. Under these conditions it is impossible to construct an index number which is unequivocal. Finally, even if this study be successful in removing most of the distorting effect of changes in relative prices during the years under investigation, it has only succeeded in artificially freezing prices into the relationship they had to each other for the base period (January, 1932). The cost-relationship formulas derived by this study are predicated upon this particular status of relative prices and/or relative quantities.

In addition to removing from the data the effect of changes in

material prices it was also found necessary to correct the cost data for fluctuations in wage rates. Great changes occurred in wage rates during this three-year period and the effect of these changes upon cost was striking. Wage cuts, late in coming to the firm, hurried fast on one another during 1932. The average hourly rate for direct labor was

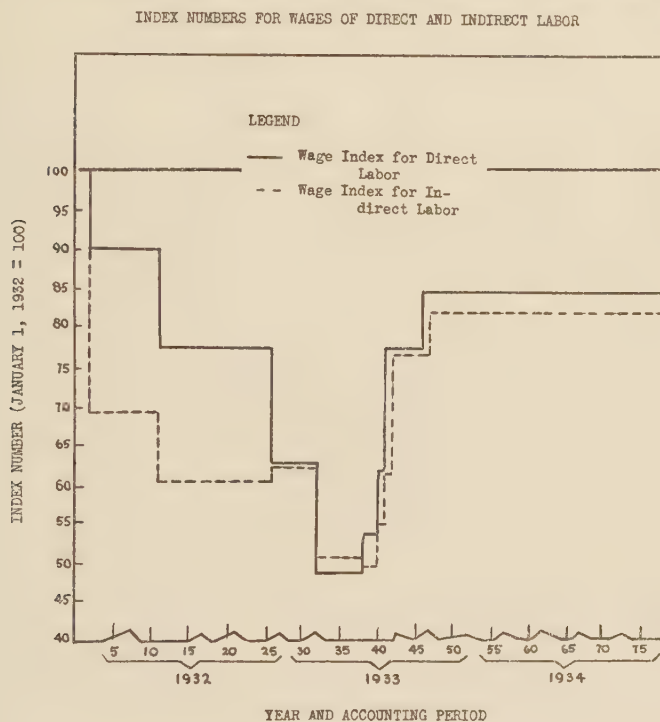


FIG. 1

forced down by April, 1933, to 48.6 per cent of its January, 1932, level. Wages for indirect labor followed a similar course. However, almost immediately afterward, wage rates were stepped up sharply, and by July, 1933, had almost reached the January, 1932, level (see Fig. 1).

The correction for the effect of these sudden changes was accomplished by computing a separate index number for each direct and indirect labor department. It was thought advisable to compute these

individual index numbers because: (1) there was some diversity in wage-rate fluctuations between departments; (2) the time lag adjustments, which were required in order to match costs with output, were different for various departments; and (3) many of the labor departments were to be made the subject of a special study, and this study would be more accurate if a specific, rather than an average, index were used for correction.

The data used for computing these indexes were the hourly wage rates found on pay-roll cards. A simple arithmetic average rate was computed for each direct and indirect labor department, and expressed as a relative with the rate of January, 1932, as the base. In addition to these separate departmental wage indexes a general index for direct and for indirect labor was computed by the same method. These index numbers are plotted in Figure 1 to show the extent of wage-rate fluctuations.

In order to correct the data for time lag (i.e., for a varying lapse of time which intervened between the recording of output) a reallocation of expenses to accounting periods was found necessary. The process-cost system used by this firm recorded output in the accounting period during which the product passed from the cabinet room to the finishing room. The majority of costs, on the other hand, were recorded in the accounting period during which they were applied to goods in production. Since goods usually remained in process at least two weeks, costs were frequently recorded in a different period from the one in which the output to which they contributed was recorded. Thus, comparison of costs as recorded with output as recorded would fail to associate output with the costs actually incurred in its production. Throughout the production cycle (which averaged about three weeks) costs of the departments which worked on the product prior to its arrival in the finishing room were certain to be recorded at an earlier date, and might be recorded in an earlier period than was the output to which they contributed.

In view of the seriousness of this time discrepancy it was clear that a mere comparison of recorded costs with recorded output would have failed to associate output with the costs actually incurred in bringing it into existence. Either costs or output would have to be shifted to their proper period, in order that the two could be matched up. But

owing to the lack of departmental production records output could not be reassigned to previous periods. Consequently, costs had to be corrected for this discrepancy. In making this cost correction two kinds of problems presented themselves. The first concerned methods of determining the amount of lag or lead for each cost item. The second centered about methods of adjusting cost for the lead thus determined. The treatment accorded direct departments was somewhat different from that given indirect departments; hence they are discussed separately.

As an *a priori* method of determining the extent of time lag for costs suggestions were elicited from production and accounting executives and were supplemented by an examination of accounting routines. Two kinds of discrepancy were ferreted out: (1) the lapse of time between the recording of the cost and the performance of the service and (2) the lapse of time between the performance of the cost service and the recording of output.

These *a priori* hypotheses regarding cost lag were then subjected to three kinds of empirical tests: production schedule analysis, time series comparison, and multiple correlation analysis. Production schedule analysis was the only method used for direct labor departments; time series analysis was used primarily for indirect labor departments; and multiple correlation analysis was applied only to tools and to finishing materials.

In the first of these empirical tests an informal analysis of production schedules by the planning department determined the average number of days an order remained in each department. The cumulative figure represented the number of days by which costs in that department anticipated the recording of output; then these figures were transformed into percentages of the ten-day accounting period. This percentage indicated the proportion of the costs of this department which was to be shifted forward or back one period. This procedure, however, was open to several objections: (1) the averages, upon which the whole scheme was based, covered up a great deal of variability; (2) differences in the type and quality of product produced during a particular period caused considerable variation in the operating schedule which was not reflected in the schedule of adjustments; and (3) complexity introduced at this point in the analysis would have to be

cleared up when results were applied to forecasting costs. Nevertheless, this rough method was used to test suggested lags and leads of direct labor departments.

Time series analysis (the second of the empirical tests) was carried out usually by means of time charts and scatter-diagrams. For constructing time charts the cost elements were first corrected for a price change and reduced to a per-unit basis. Then these figures were plotted as a time series and compared over a light box with output (also plotted as a time series). In some cases a scatter-diagram was drawn relating output with a cost item which had been corrected for a suggested lag or lead. This was then compared with a scatter-diagram relating output with the uncorrected values of the cost element. By these informal statistical devices various deductively suggested lags and leads were tested for optimum fit.

On the basis of this test it was possible to group overhead costs into five categories according to their indicated approximate lag or lead-over-output reportings: (1) no indication; (2) no lag or lead; (3) lag of less than one period; (4) lead of less than one period; (5) lead of one period or more. For numerous overhead items no adjustment for lag or lead seemed advisable. Only when empirical and a priori considerations clearly indicated the need for a lag or lead adjustment was one made.

In closing it should be reiterated that this chapter is but a summary description of the methods actually used⁷ to free the cost-output relationship from the distorting effects of irrelevant variables. Moreover, the correction devices used in this study are, to some extent, peculiar to this case. Nevertheless, the problem of correcting the data for various types of distortion will be encountered in any short-period cost study.

DISTORTION CAUSED BY CERTAIN FACTORS ALLOWED FOR BY MULTIPLE CORRELATION ANALYSIS

Finally, when the consequences of irrelevant factors could not be prevented and when their effects could not be measured independently, then, in order to remove them from the data, a third general method

⁷ For a more complete description of methods used and a discussion of alternative procedures see the Appendix of the author's dissertation previously cited.

for preventing cost distortion was employed. These recalcitrant factors (if important and susceptible to statistical treatment) were introduced as independent variables in a multiple correlation analysis. Thus their effects were removed by studying the relationship of cost to one independent factor (output) while holding the other independent variables constant at their means. This method of allowing for the effects of cost influences is treated in the next chapter, where quantitative analysis of average cost behavior is discussed.

CHAPTER IV

METHODS OF ANALYZING COST DATA

Up to this point the difficulties encountered in preparing the cost data for analysis have been discussed. In the present chapter the problems involved in analyzing these rectified cost data are considered. These problems center upon the determination of the relationship between cost and the various factors which affect its behavior. For the purpose of the present discussion they may be grouped under the following major headings:

- A. Selection of technique for analyzing cost behavior
- B. Selection of cost constituents to be analyzed
- C. Selection of accounting periods for analysis
- D. Choice of factors affecting cost behavior
- E. Methods of deriving marginal cost

Obviously, all these problems are actually too closely interrelated to be considered separately. Decisions as to the kind of technique to be used depend upon the independent variables available, the nature of the sample, etc. For clarity in presentation, however, these problems have been separately considered as listed above.¹

SELECTION OF TECHNIQUE FOR ANALYZING COST BEHAVIOR

In order to secure usable knowledge of average and marginal cost behavior the following definite kinds of information were required:

1. Some definite measure of the net effect of several important cost influences upon cost behavior. Such a measure should take the form of a curve or a schedule of per-unit or of total cost at a series of values for each important factor affecting cost. In such form it is directly usable for setting standards and making forecasts of average and of total cost, and also for determining marginal cost.
2. Some indication of the combined importance of the effect of these causal factors upon cost. Such a measure is highly serviceable in

¹ A brief summary of the methodology is found in this chapter. The reader who is interested in a more complete description and evaluation of the methods employed will find such a discussion in the author's dissertation, "A Statistical Examination of the Behavior of Average and Marginal Costs," Appendix B.

determining the limits of usefulness of cost standards, cost predictions, and of marginal costs computed on the basis of the foregoing relationships.

3. Some measure of the reliance which can be placed upon the curves or schedules as representations of cost behavior. This kind of measure is of particular importance in the case of the curve showing the relationship between output and cost, since this curve is the basis for the determination of the marginal cost curve.

Several alternative techniques for analyzing cost behavior were considered in order to ascertain which would give the required information most completely and most economically. The most important of these methods are:

1. Estimates of cost behavior on the basis of a classification of costs according to their variability as indicated by a priori notions
2. Multiple tabulation of actual cost data, sorted on the basis of the values of the various factors which influence cost
3. Joint correlation analysis of the effects of factors which operate together
4. Mathematical multiple correlation analysis of the effects of influential cost factors
5. Graphic multiple correlation analysis of the effects of important cost factors

From the foregoing methods graphic multiple correlation analysis was chosen as the most suitable for this project. This choice was made despite the fact that the graphic technique is inferior to the others in certain respects: It is more laborious than the method of estimates; it is less satisfactory than joint correlation if joint relation with the dependent variable exists; and it is inferior to mathematical multiple correlation both in objectivity and in the definiteness of its error formulas. Nevertheless, it was preferred because of its economy (as compared with all except the estimates method) and flexibility, and because the regression curves and error formulas which are used as implements of analysis also serve admirably as methods of presenting the average and total cost findings and lead conveniently to the derivation of marginal cost curves. The well-known Bean-Ezekiel technique of graphic multiple correlation analysis was employed. It is fully described by Ezekiel, in *Methods of Correlation Analysis* (New York: John Wiley & Sons, Inc., 1930), chapter xvi.

SELECTION OF COST CONSTITUENTS TO BE ANALYZED

In deciding which cost constituents should be thus analyzed the choice lay between limiting the study to an analysis of corrected combined cost alone, or broadening the scope of the investigation to include an examination of the various constituents of combined cost.

The decision depended upon whether or not the objectives of this investigation could be realized short of the more detailed study. These objectives were: (1) exploration of methodology; (2) development of a practical procedure; (3) investigation of average cost behavior for the purpose of (*a*) determining flexible cost standards, (*b*) improving cost forecasts; and (4) determination of marginal costs. Despite the fact that combined cost is of dominant importance, both for managerial control and for marginal cost determination, it was clear that nothing short of detailed multiple correlation analysis of cost elements would bring an approximate realization of the objectives mentioned above.

The following definite contributions to methodology resulted from the analysis of the cost constituents: (1) Detailed correction of the data for cost elements was made necessary, thereby developing some refinements in correction technique. (2) Additional independent variables were introduced which were of peculiar importance to a particular cost element, but of relatively little significance for the analysis of combined cost. (3) Methods of allocating and combining cost elements were developed in an effort to apply the results of these detailed studies to improving cost standards and cost forecasts.² (4) A rough test of the value of correcting data for rate changes and for time lag was developed by means of the analysis of uncorrected combined cost.

As a result of these additional detailed analyses flexible cost standards were determined more accurately and were made more useful to management. Accuracy was increased by the employment of independent variables which were thought to be of peculiar significance to the specific cost being analyzed. The usefulness of cost standards was enhanced because costs are more easily controllable in detail than in total and causes for departure can be determined more accurately and responsibility for adherence to standards assigned more definitely.

The accuracy and usefulness of cost forecasts were also increased

² For a discussion of this contribution see chap. viii, "Practical Application of Findings."

by the addition of detailed analyses. Accuracy was increased, partly because prediction was based upon a somewhat better understanding of the causes of cost behavior and a quantitative measure of the past effect of various factors upon cost constituents, and partly because it was safer and easier to make a priori allowances for the effect of changed conditions upon individual cost constituents than upon their total. The usefulness of cost forecasts was extended because they were now available in more controllable detail and also in a form enabling allocation of estimated cost more specifically to particular articles or output lots.

Estimates of marginal cost presumably could be assigned more definitely to a particular article or a particular order³ by using the marginal curves of cost elements, instead of assigning combined marginal cost to the article on the basis of its old warehouse value alone. This procedure, however, assumes the separability of these cost constituents and presupposes considerable accuracy in the estimates of their marginal cost.

For each of the following elements of cost a separate multiple correlation analysis and marginal cost derivation were made:

- | | |
|---------------------------------|-------------------------------|
| 1. Corrected combined cost | 7. Inspection labor cost |
| 2. Uncorrected combined cost | 8. Yard labor cost |
| 3. Direct material cost | 9. Power and watch labor cost |
| 4. Direct labor cost | 10. Indirect materials |
| 5. Combined indirect cost | 11. Power |
| 6. Combined indirect labor cost | 12. Maintenance |

SELECTION OF ACCOUNTING PERIODS FOR ANALYSIS

Data for all the available accounting periods could not be used in the analysis, partly because of defects in the records, and partly because of the limits placed by the graphic method itself upon the number of observations which could be efficiently analyzed. It was decided that a

³ Marginal cost can be assigned more definitely, but not necessarily more accurately. This allocation, of course, only arbitrarily cuts the Gordian knot of a problem which strictly is soluble only when it is possible to vary the proportions of the cost elements entering into the product and to determine the incremental cost of alternative combinations. All allocations of joint cost short of this rather theoretical solution are arbitrary. For a more complete discussion see the author's dissertation, *op. cit.*, chap. x.

three-year period would provide a sufficient number of observations. The years 1932-34, inclusive, which were chosen, had certain advantages: (1) the short-run rather than long-run character of cost adjustments during this period; (2) the pronounced variability in all operating conditions; and (3) the recency of these three years.

Of the seventy-eight two-week accounting periods in these three years, thirty-one had to be discarded because of various types of errors and omissions in the data. The remaining forty-seven accounting periods were employed in the analysis of the following major items of cost:

- | | |
|--|----------------------------------|
| 1. Corrected combined cost per unit ⁴ | 3. Direct labor cost per unit* |
| 2. Uncorrected combined cost per unit* | 4. Direct material cost per unit |
| | 5. All indirect cost per unit |
| | 6. Indirect labor cost per unit* |

For the analysis of subsidiary items of overhead cost a smaller sample of twenty-five picked accounting periods was used. The principal criteria used in the selection of this sample were: (1) that satisfactory data be available and (2) that the values of the independent variables be fairly evenly distributed over their range. The sample thus chosen was employed in the following subsidiary analyses:

- | | |
|---|---------------------------------------|
| 1. Inspection labor cost (as a total)* | 4. Indirect material cost (per unit)* |
| 2. Power and watch labor cost (as a total)· | 5. Power cost (per unit)* |
| 3. Yard labor cost (as a total)* | 6. Maintenance cost (per unit)* |

CHOICE OF FACTORS AFFECTING COST BEHAVIOR

The decision being made to use graphic multiple correlation techniques for analyzing the foregoing cost items, it was necessary, first, to choose as independent variables those factors which were most important as influences upon cost behavior and, second, to select the most suitable statistical series with which to represent these factors.

The choice of cost factors was made on the basis of the following criteria: (1) the factor should have a significant influence upon cost; (2) the factor should represent a cost influence which is distinct, that is, which is not to any important degree already included in some other

⁴ Costs are reduced to averages per dollar of old warehouse value of output. The starred items are discussed briefly in this abstract and more fully in the author's dissertation, *ibid.*, chap. vii and Appendix B.

independent variable; (3) the factor should be susceptible to statistical measurement.

Not all the factors which were used satisfied all the foregoing conditions, but it would have been difficult to find factors which would. The factors selected on the basis of these criteria were:

X_2 —Output	X_7 —Instability of labor force
X_3 —Size of production order	X_8 —Quality of material
X_4 —Unfamiliarity of patterns	X_9 —Receipts of lumber
X_5 —Change in output from previous period	X_{10} —Fuel outlay

The second step in selecting the independent variables was to choose statistical series for representing the factors listed. Output was measured by old warehouse value; size of production order, by the average number of pieces per manufacturing lot; unfamiliarity of patterns, by the number of new styles produced for the first time in the accounting period in question. Change in output from the previous period was represented by two alternative statistical series. The series used in most of the analyses were increases or decreases in output from the previous period measured in dollars of old warehouse value. In analyzing direct labor and direct materials, however, percentage increases and decreases were used. Instability of the labor force was measured by the ratio of the number of additions to the force to the total number of men on the pay-roll; quality of materials, by the ratio of the standard cost of output to the old warehouse value of output;⁵ receipts of lumber, by the number of board feet unloaded; and fuel outlay, by the dollars of expenditure for coal.

Some of these independent variables were found to be of greater significance for certain cost items than for others. Table II shows the set of independent factors employed in the analysis of each cost item.

METHODS OF DERIVING MARGINAL COST

These multiple correlation analyses of average and total cost provided rectified data for the determination of marginal cost. By holding constant the effect of other variables the pure relationship between cost and each of the independent variables was brought out. Thus the de-

⁵ This ratio is not a pure index of the quality of materials since standard cost differs from old warehouse value in other respects.

termination of the increment in cost which was associated with a change in one of these cost conditions was made easier. Although the incremental cost of each of the several independent factors may be of practical significance, the marginal cost of increases in the quantity of output is easily of greatest importance.⁶

In deriving marginal cost curves from these data one of two alternative general procedures could be followed. The data could be studied either in the form of totals or in the form of averages. In the first procedure a function was fitted to the total cost residuals secured from the net regression chart for output. This total cost function was then differentiated, and its first derivative taken as the expression for estimated marginal cost.

Three techniques were employed in fitting a function to these total cost residuals: (1) visual, (2) selected points, and (3) least squares. The visual method was used as a part of the graphic correlation analysis in the case of those items whose data had been kept in terms of totals (yard, inspection, and power labor costs). The method of selected points provided a short-cut technique for converting these flexible visually fitted curves into mathematical expressions which could be conveniently differentiated to secure an equation for marginal cost. For example, for yard labor cost the resulting equations, together with a series of substituted values, are shown in Table XIX. The method of least squares was used for combined cost. By this third technique both a straight line and a parabola were fitted to the total cost residuals of combined cost. The results of this experiment are shown in Figure 4 and Table IV.

As an alternative to this total cost procedure, a second general technique was developed which derived marginal cost by manipulating a function fitted directly to the individual average cost residuals. The total cost function was secured in this case by transforming the fitted average cost function rather than by transforming the individual residuals to total cost residuals and then fitting a function to these.

The basic procedure was very simple. First, the net relationship

⁶ Margins of combined cost for size of production order (X_3), number of new styles (X_4), change in output (X_5), and labor turnover (X_7) were computed in addition to the more familiar output margin (in this study called marginal cost). In investigating the components of combined cost, however, only the output margin was studied.

between output and average cost per unit was determined. This relationship was recorded either as a curve which had been visually fitted to regularly spaced group averages found in the final regression chart, or as an equation which had been mathematically fitted to these group averages (or to the individual observations). Second, the resulting average cost function (curve or equation) was converted into a function which expressed these cost values as totals, rather than as averages per unit of output. Third, the first differences of this total cost series were taken. If the relationship was expressed as an equation, then the first derivative of the total cost equation gave the equation for the marginal cost curve. If, on the other hand, total cost was represented as a curve or a table of values, then its first difference series had to be smoothed by means of a moving average and by inspection to secure the required marginal cost curve.

Four variants of this fundamental average cost procedure may be distinguished primarily on the basis of the methods of curve fitting. They may be called for ready reference:

1. The "first differences method," in which no curve was fitted to the average cost data
2. The "graphic method," in which curves were fitted visually and then differentiated by tabular methods
3. The "derivative method," in which the least-squares method was used for curve fitting and the calculus for differentiating
4. The "graphic-derivative method," in which the visually fitted curves were transformed into equations by the method of selected points and differentiated by the calculus

In the method of first differences the relationship between average cost and output was recorded and manipulated in the form of a series of regularly spaced group averages of per-unit cost. Next, the series of interval averages of per-unit cost was converted into a series of total cost values by multiplying each item of the per-unit cost series by its corresponding output value. And, finally, the first differences of the resulting total cost series were then computed and smoothed to obtain the marginal cost curve. Figure 5 shows the results obtained by this method for combined cost.

The graphic procedure is somewhat simpler than the method of first differences. Here the visually fitted net regression curve of average

cost on output represents the relationship between per-unit cost and the rate of production. The per-unit cost series (regularly spaced readings from the curve) was converted into a total cost series by multiplying each member by the output at which this curve reading was taken. The first differences of the resulting total cost series were then smoothed to yield the marginal cost curve. The resulting curve is shown in Figure 5.

The method of derivatives differed from those just discussed in that the relationship between output and average cost per unit was expressed in the form of an equation rather than as a series or a curve. This equation was obtained by the method of least squares, from the per-unit cost observations found in the final net regression chart of average cost and output. When this average cost equation had been converted into the total cost equation, its first derivative was taken as the equation for the marginal cost curve. Results secured by this method are shown in Figure 5.

The hybrid method combines the first steps of the graphic procedure with the last steps of the derivative technique. This method took as the average cost curve the graphically fitted final net regression curve of per-unit cost on output. To this curve an equation was fitted by the method of selected points. The average cost equation thus secured was then transformed into an equation for total cost, whose first derivative was the mathematical expression for the marginal cost curve. This hybrid method was used as one procedure for deriving the marginal cost of most of the components of combined cost.

An evaluation of each of the foregoing procedures, based upon a comparison of the results secured by each method, is found in the chapter dealing with the marginal curve for combined cost (chap. vii). A complete comparative description of these procedures is also found in the author's dissertation previously cited.⁷

⁷ Chap. viii and Appendix B.

CHAPTER V

RELIABILITY OF FINDINGS

As a preface to the presentation of the specific findings of this study, certain general limitations upon the reliance which can be placed in these results should be stated. These limitations apply to both average and marginal cost behavior. They also apply both to the accuracy with which the cost formulas represent the actual cost behavior of the period analyzed and, more especially, to the dependability of the formulas as a forecast of future cost behavior. The accuracy and stability of these cost relationships are of considerable interest since their reliability determines, to a great extent, the managerial usefulness of the findings of this study. Hence it was thought advisable to summarize and evaluate in one chapter the various limitations so that they could be readily kept in mind in considering the later chapters of this study, which present the findings of this investigation into cost behavior.

These limitations will be discussed under the following four heads:

1. General assumptions regarding cost behavior
2. Defects in the data
3. Imperfections in the technique of analysis
4. Limited applicability of the error formulas developed

GENERAL ASSUMPTIONS REGARDING COST BEHAVIOR

Certain general assumptions regarding cost behavior have been tacitly made in the course of this project. In the first place, the cost function of the firm studied is assumed to be sufficiently separable from the demand function to permit its independent analysis. This assumption appears to be substantially valid, for the functional relationship between manufacturing cost and its causal factors does not seem to be affected to a quantitatively significant degree by the shape and position of the demand schedules for the products made by this firm.¹

¹ If the output of this concern varied with fluctuations in general business, then the prices of those input items which responded sensitively to business conditions would tend to be positively correlated with output. In that event the undeflated money cost curves would be related indirectly to the quantity of the firm's product which was demanded, but not to the shape or the position of these demand schedules themselves.

A second assumption, namely, that the cost curves for this firm are sufficiently reversible and sufficiently continuous to be portrayed as two-direction curves or bands, is more questionable. Cost adjustments required to increase output from x to $(x + 1)$ output are not the exact reverse of those required to reduce output from $(x + 1)$ back to x . A rise in output which is followed by a drop to its former level is unlikely to return cost immediately to its previous figure. An attempt has been made to allow for this distortion by introducing, as an independent correlation variable, the amount and direction of change in output from that of the previous period. This correction, however, is too simple to allow for the complex patterns of change. Some account should also be taken of the number and amount of successive decreases or increases in output which preceded the current adjustment and, if possible, of the length of time at which it was expected that production would remain at the new level.

Doubt also attaches to the concept of the continuity of the cost functions. A smoothed curve is a statistical approach to a reality which could probably be more correctly represented by a staircase of discrete steps of cost adjustment. However, no great violence appears to have been done by this conventional assumption of continuity.

In the third place, it is assumed that the proportion in which the cost constituents are combined is affected by changes in the rate of production, size of lot, new styles, etc., but that it is not affected by changes in the relative prices of constituents. Examination of the regression curves of the components shown in Figures 9-15 of Appendix A indicates that the proportion of cost elements does vary with the rate of output and with the other independent variables. The second part of the assumption would appear to be more questionable. But, for the breakdown of cost used in this analysis, the proportion actually seems not to be affected by changes in the relative prices of the factors, within their observed range.

The technology of production is fairly rigid in the short run in a firm so highly mechanized as the one studied. Hence the resources are not very adequate substitutes for one another, since significant substitutions among the main categories of cost usually involve considerable rearrangement of the techniques of production. For the period studied in this investigation the technical coefficients of production at any

given level of output, order size, etc., have probably remained fairly stable regardless of the considerable shifts which took place in relative prices. Substitution caused by changed relative prices has probably been confined mainly to shifts within the classifications of cost used in this study, e.g., within the general category of indirect materials, and has not extended to shifts among these classes of cost. With continued stability of technology it appears unlikely that the findings for individual cost components will be invalidated for future use by an increase in substitution, since shifts in relative prices greater than those which occurred during the period of study seem improbable. Thus it appears that, although some substitution will to a certain degree limit the significance of the separate analysis of cost components, it will not seriously damage its usefulness.

The fourth assumption is that the underlying causes for the relationships between cost and the several independent variables have remained essentially the same during the lapse of time covered by this study. This assumption appears to be fairly correct for the particular period analyzed, since a series of years was selected during which technology, production routines, and management personnel remained about the same. The general stability of the underlying conditions in these three years is in striking contrast to the marked variability in the intensity of use of the fixed factors and in the prices of certain of the variable input ingredients. This stability of plant scale and other fundamental conditions makes the observed behavior of cost in response to short-period changes in output fair approximations to the theoretical neoclassical cost functions developed from the "timeless" assumptions of static economics.

The cost formulas determined on the basis of past cost behavior in this period of exceptional stability in production techniques, accompanied by exceptional instability in output, size of lot, and the like, and in input prices, cannot be expected adequately to portray the cost relationships of future periods. If underlying conditions change rapidly in a manner which cannot be allowed for, then the results of this study will be of little direct value to management in setting standards, making forecasts, and deciding upon price and output policy; but it will still be useful as indicating a procedure for dealing with the changed circumstances.

The fact that these relationships were determined for data representing a depression period throws considerable doubt upon the value of these findings in making forecasts for normal periods. Cost behavior for depression years may be considerably different for several reasons. First, the altered morale of the labor force doubtless affects cost. On the one hand, fear of unemployment tends to drive men to attempt greater accomplishment, while the psychological accompaniments of a depression may unfavorably affect the productive capacity of the more sensitive workers. The realization of the imminence of one of the periodic shutdowns may cause some loafing at the tail end of a run. Both these tendencies would alter cost relationships. Second, selection of top-grade workmen through selective layoff should result in varying reductions of labor cost covering all departments.² Third, reduced schedules narrow the range of actual output levels, and thus fail to provide information as to cost relationships for high rates of production activity. Fourth, managerial skimping on postponable expenses may have altered temporarily the proportion of such items of cost.

The strong probability that changes in the cost relationships will occur in the future for the firm studied indicates the need for devices for adjusting the formulas for some changes and for recomputing them when this adjustment is no longer reliable. In applying the cost relationships developed in this study it should be recognized (1) that their dependability is limited by some inevitable changes in cost conditions during the period covered by the analysis and (2) that the unreliability of these results for prediction purposes increases with the magnitude of such changes in the future and with the length of time which elapses between the analysis and the forecast.

A closely associated assumption is that changes in managerial control of costs have not distorted the cost relationships under analysis. It is unreasonable to assume that management is omniscient and errorless; but it is not unreasonable to assume that its failings remain fairly constant for the period under investigation. The truth most likely is that the character and degree of managerial control over cost probably

² Professor Viner suggests that the workers selected for retention are likely to be those having high output during good periods. These men, because of their greater psychological instability, may be the very ones whose capacity to produce is most curtailed by the psychological effects of bad times.

have not remained entirely constant. Hence this assumption of constant managerial efficiency is not entirely accurate.

The error in the foregoing assumption affects the reliability of predictions more than it does the accuracy of the statement of cost relationships during the period studied. Managerial changes far more drastic than those which occurred during this period may transpire in the future to curtail substantially the usefulness of these particular results as standards and forecasts.

It is also assumed that old warehouse value is a satisfactory measure of output. To be satisfactory this measure should have the following characteristics: (1) It should not reflect changes in physical inputs per unit of output caused by shifts in the rate of production. In other words, it should measure output rather than input if the two do not vary together. (2) It should not reflect changes in the value of physical output units caused by shifts in desirability or by changes in the price level.

As far as the investigator could ascertain, old warehouse value seems to be fairly satisfactory in each of these two respects. One other condition must be satisfied, however, if this measure is to be entirely adequate. The proportions of the models making up output must not seriously affect the combination of cost components. It must be true that the proportions between different models remain constant, that changes in the proportions average out, or that the same cost components are used in approximately the same way regardless of the character of the particular piece of output being made. The partial realization of all three of the foregoing conditions probably eliminates most of the error which could come from this source.

DEFECTS IN THE DATA

A second general type of limitation upon the reliability of the findings of this study consists of defects in the data. These imperfections have already been discussed in connection with attempts to correct them. It is therefore sufficient, at this point, to recall them to the attention of the reader in order that he may be on his guard against the residuum of error remaining after their incomplete correction. Limitations of this type are not peculiar to this investigation alone, but will be found in any study of a similar nature. Serious though these limita-

tions are, they prove upon examination less damaging than their mere mention would indicate. Many of them may be randomly distributed with respect to the cost functions and hence may not prove distorting. The principal defects in the data may be classified under the following general headings:

1. Measurement of the phenomena studied was often indirect and frequently inexact. Although the units finally selected for measuring cost and the various independent variables seemed to be the best available, they still were not all that could be desired. The various adjustments of the reported cost figures removed most of the error associated with accounting routines, but here, too, correction was not perfect.

2. The manner of recording cost and output may have covered up significant variations. Daily fluctuations in cost, output, etc. were lost by totaling or averaging for the two-week accounting periods. Also the combining of varied types of product by using old warehouse value as a common denominator may have hidden cost variations arising from the diversity of output. This imperfection, however, is probably not as serious as it appears since the assortment of output is actually fairly constant and the processes are about the same for all units of output.

3. Correction of the cost data for irrelevant cost influences was not complete. These corrections were, however, as refined as will prove practical in commercial investigations and were probably sufficiently adequate for determining the general character of the cost functions.

4. The representativeness of the sample may be questioned: It was small and covered three depression years. The sample, however, was as large as was manageable with the technique chosen, and the advantages of the particular three-year period (as to recency, stability, and multiplicity of methodological problems) appeared to outweigh its cyclical disadvantage.

IMPERFECTIONS IN THE TECHNIQUE OF ANALYSIS

A third group of general limitations grows out of the methods of analysis. These limitations have already been discussed; consequently, they will only be listed here as a reminder of the extent to which they impair the reliability of the results of this study.

1. The partial correlation measures of cost behavior often embrace

more territory than they purport to. One of the properties of the net regression curve is that it ascribes to any specific independent variable, such as output, not merely the behavior of the cost due to this variable alone, but also the variation which is due to such other independent variables as are correlated with this variable and have not been separately considered in the study.³

2. There were some important cost factors which were not separately included in this study. It is possible, however, that the omitted factors were randomly distributed with respect to the main cost relationships; and it is improbable that their inclusion would have had a pronounced effect upon the cost-output relationship.

3. The curve fitted by the graphic method may not represent accurately the observations upon which it is based. Visual fitting lacks, to some extent, a desirable objectivity since so much is left to the personal judgment of the investigator. Moreover, the ease with which complex curves can be fitted is a temptation for their too generous use.

4. Marginal cost curves derived from flexible, visually fitted average cost curves frequently exhibit spurious and illogical fluctuations. This is due to the fact that a slight flexion of the average cost curve causes a magnified fluctuation in estimated marginal cost. It is often the case that only the general level and direction of marginal cost curves derived by this method are significant. On the other hand, those derived from total cost observations are more reliable since marginal cost is a simple function of the total cost curve (its slope) and hence unwarranted fluctuations in marginal cost can be more easily avoided.

LIMITED APPLICABILITY OF THE ERROR FORMULAS DEVELOPED

A fourth type of general limitation concerns the confidence which can be placed in the formulas for standard error. These formulas purport to state in precise terms the degree of reliability of the results obtained. The use of these formulas, however, is based upon certain assumptions not completely realized in this case.

1. A random sample from an unchanging universe is assumed.⁴ It is

³ Mordecai Ezekiel, *Methods of Correlation Analysis* (New York: John Wiley & Sons, Inc., 1930), p. 169.

⁴ H. R. Tolley, "Economic Data from the Sampling Point of View," *Journal of the American Statistical Association*, XXIV (1929), 70 (suppl.).

clear that our sample is not a random one. The major sample of forty-seven accounting periods represents a fairly complete coverage of a presumably typical three-year period. If the cost behavior was typical during this period, then the sample is representative. The minor sample of twenty-five cases is a "purposive" sample⁵ selected to cover the range of the independent variables. Not only is the sample not random but our universe is clearly not altogether unchanging.

Although the sample is not random, it may, nevertheless, be fairly representative of cost behavior during a period remarkably free from changes in the underlying cost conditions. Whether or not the universe is unchanging depends upon the meaning given the term "universe." In a time series study such as this, the universe could be conceived of as an imaginary one in which operating conditions, prices, etc. remained roughly within the same range as during the three years studied. With this view of the matter the error formulas would indicate whether or not successive samples of this type taken from this assumed universe would show that the independent factors did affect cost approximately as specified in the past period which was analyzed.⁶ It appears therefore that, although the assumption of random sampling from an unchanging universe is not satisfied by the conditions of this study, the error formulas may nevertheless be useful, even if not completely satisfactory in judging reliability.

2. Normal frequency distribution of the data is assumed by the least-squares error formulas. The validity of this assumption may be tested by the examination of the plotted observations. Such inspection indi-

⁵ The difficulty of finding a random sample for economic data led Yule to develop the "stratified sample" and Bowley to work out a "purposive sample." The small sample of accounting periods which was used for the subsidiary correlation analyses is a sort of cross between these two types of samples. It attempts to cover the range of all independent variables, but was selected with certain definite ends in view. It has already been noted that this selection, with respect to the values of the dependent variables, does not tend to alter the regression curves, but does somewhat change the correlation coefficient.

⁶ Some serial correlation was noted in the corrected data, since successive accounting periods tended to have deviations of the same sign from fitted curves. This common characteristic of time series data makes error formulas tend to overstate the probability of recurrence of observed relationships in other samples taken from the same universe.

cates that normal frequency distribution was approximated in a large part of the data used for this study.⁷

3. Error formulas computed by graphic methods are assumed to have a significance parallel to that of those computed by classical least-squares methods. The graphically determined standard error of estimate is fairly well established. But the graphic methods suggested by Ezekiel for computing the standard error of a curve are still provisional. The general principles upon which these formulas are based, however, appear reasonable, and the results secured have stood up well under extensive experimental testing. Moreover, it should also be pointed out that these graphic error formulas were supplemented in this study by other criteria of significance computed by orthodox methods.

In general, it appears that the assumptions underlying these error formulas are not as far from the actual conditions of the investigation as might at first appear. A representative sample of a practically static and fairly typical period approximates the ideal of a random sample of a changeless universe. Almost normal distribution was found in the case of many of the variables. And graphic error formulas, although provisional and not exactly parallel in significance to least-squares formulas, appeared useful in indicating the significance of observed cost relationships, especially when supplemented by other criteria of significance.

In conclusion, a survey of the various limitations seems to indicate that: (1) a fair amount of faith can be placed in the general shape and location of most of the average and total cost functions; (2) less confidence can be placed in the marginal cost curves which were derived from average cost curves than in those computed from total cost functions; and (3) the dependability of the cost relationships for use in the future diminishes with the amount of change in cost conditions which takes place in the time which elapses.

⁷ Figs. 2 and 9 clearly indicate that two of the independent factors (X_3 and X_5) have approximately normal distribution, that two others (X_2 and X_8) have a flat-topped, fairly symmetrical distribution, and that still two others (X_4 and X_7) have extremely irregular distribution. The distribution of most cost items (dependent variables) seems to be skewed toward low costs.

CHAPTER VI

BEHAVIOR OF AVERAGE COMBINED COST

Knowledge of the behavior of average cost is useful to management for several practical purposes:

1. Cost control can be made fairer and more effective. Cost standards are made flexible so that they allow automatically for operating conditions over which production management has no control and for which it should therefore not be held responsible. Standards thus set are alibi-proof, for, if current average cost departs from these adjusted standards, the factory management alone can be held accountable.

2. Budgeting of expenses and profits can be made more accurate and flexible. Forecasts of costs can be made with greater precision and can be adjusted more gradually and more certainly for changed operating conditions when they are based upon knowledge of the actual effect of important cost influences. Flexible budgets of this character are more useful as control devices and more reliable as a basis for profit predictions than are fixed budgets. With this basic knowledge of average cost behavior the budget can be made alive and adaptable; it can be tailored to fit the actual operating conditions of the period and speedily remodeled as these conditions change.

3. Evaluation of cost-reduction policies is made more objective and accurate. A proposal to take steps to diminish the number of new styles, for instance, could be better appraised if the average net effect of the introduction of a new model had been measured.

Attainment of these practical objectives requires, first of all, a careful measurement of the effects of various operating conditions upon average costs.¹ The first section of this chapter presents a statement of the observed character of the relationship of corrected combined cost to each of several important cost influences, together with an ap-

¹ The neoclassical theorists talk about other cost factors in addition to output, but concentrate their attention upon the effects of output and dispose of the other variables by the assumption that "all other things remain equal." The present analysis recognizes that, in fact, these other factors do not remain the same and that it is important to measure the actual effects upon cost of their variation.

praisal of the degree of reliability of these functions and a measurement of the degree of closeness of the relationships.² In the second section of this chapter the behavior of uncorrected combined cost is presented with a view to determining whether or not the elaborate correction of the data was worth while as judged by its effect upon average cost results.

In describing the various findings for average cost behavior several general types of results of the graphic multiple correlation analyses may be noted.

1. Statements of the relationship between cost and the several independent factors
 - a) Net regression curves showing cost at various values of the several independent factors
 - b) Tables of estimated cost under various operating conditions, presenting in a more understandable form the information portrayed by the regression curves
2. Measures showing the probable reliability of results
 - a) Standard error of estimate of cost
 - b) Standard error of regression curve
3. Indexes showing the closeness of the relationship between cost and those independent factors affecting cost³
 - a) Index of multiple correlation
 - b) Index of multiple determination

CORRECTED COMBINED COST

STATEMENTS OF THE RELATIONSHIP BETWEEN COST AND THE SEVERAL INDEPENDENT FACTORS

A net regression curve states the pure relationship between cost and one independent factor (such as output) while the remaining independent variables are, in effect, held constant at their means. The relationship of cost to output is stated as the average of observed cost

² A parallel analysis of several of the components of combined cost is summarized in Appendix A of this monograph. For a more complete statement of these results see the author's dissertation, "A Statistical Examination of the Behavior of Average and Marginal Costs," chap. vii and Appendix C.

³ Indexes of part correlation and of part determination were computed for direct labor cost. The experiment was not extended to other cost items because this type of result appeared to be irrelevant to the major objective of the study. These experimental computations are found in the author's dissertation, *ibid.*, Appendix B.

associated with particular levels of output; readings from this curve show the expected cost at the corresponding level of output. The relationship of cost to any other independent factor is stated as the average deviations of cost associated with various values of such factor. Curve readings for these other factors are added to or subtracted from (as the case may be) the output curve reading in order to obtain the expected cost for a particular combination of output and other factors.

It is desirable to compare the observed cost behavior indicated by each of these regression curves with the behavior which might theoretically be expected. Such a comparison, by testing the reasonableness of results and attempting to account for illogical departures, enhances the confidence with which practical application of these results can be made.

The regression curve of average combined cost per unit on output would be expected to be asymptotic with the Y -axis, then decline to the right for a time, and finally rise again. In the short run (defined as a period short enough to permit no changes in the scale of plant, but only variation in the intensity of its use) the average cost curve would theoretically be expected to be asymptotic to the Y -axis because of the irreducible minimum of expense which must be spread over an infinitely small output. As the rate of production is increased, the curve declines to the right because of the spreading of this irreducible constant cost over a larger and larger output. Eventually the decline of average fixed cost is counterbalanced by a rise of the other component of combined cost, namely, average variable cost. This increase in variable cost per unit will eventually come about through the operation of the law of diminishing returns. As more and more of a variable input factor is applied to a constant factor, the output tends to increase at a diminishing rate. As a result the variable costs per unit rise sufficiently to counteract the decline of fixed cost and result eventually in an upturn of the combined cost curve.⁴

Thus we see that the average combined cost per unit would be expected, in the short-run period, to have high values for small output, to fall at a diminishing rate as output is increased, and, finally, to rise at an increasing rate as output is still further expanded.

⁴ See Jacob Viner, "Cost and Supply Curves," *Zeitschrift für Nationalökonomie*, III, No. 1, 27.

From Figure 2 it can be seen that the observed output regression curve corresponds rather closely to expected behavior over most of its course. It has high values for small output and falls at a diminishing rate as output is increased. However, the curve fails to rise at the maximum of the range of output found for the three years studied.⁵ Investigation of records for a period of years showed that output was rarely pushed beyond the limits included in this investigation. This suggests to the investigator that under conditions of monopolistic competition, with sharply rising selling costs, a firm may not normally push production to the point at which short-run average manufacturing costs actually rise.

In examining the regression curve for size of production order the normal expectation is that an increase in the number of cases per production order will cause some reduction in the per-unit cost. This is because a constant setup time would be spread over a larger number of items with a consequent reduction of the per-unit cost of machine operations. In addition, increased familiarity with a particular assembly would tend to diminish the cost of hand operations. These tendencies would not be offset in this case by larger inventory cost, for carrying costs have not been included in the cost data.

The observed effect of increasing the size of the production order corresponds in general with the expected effect. According to the net regression curve for number of cases per production order in Figure 2, cost declines over most of the range as the size of the lot increases.⁶

The regression curve for the number of new styles produced per period (X_4) would be expected to rise continuously. Usually, the introduction of a new style affects costs unfavorably in both a direct and an

⁵ No attempt has been made in this summary to explore exhaustively the causes for the cost behavior which was found to be associated with output. Cost reduction consequent upon higher output may be due to steadier rate of flow of production through the plant, finer subdivision of task, or better morale of the force, as well as to the spreading of fixed costs.

⁶ Strangely enough, however, this curve actually rises for a brief interval in its initial stages. The explanation for this illogical initial rise is difficult to find. Apparently it is caused in part by incomplete removal of irrelevant cost influences from the corrected data (the curve for uncorrected cost rose throughout its course) and in part by too flexible curve fitting.

indirect manner. Direct increases in cost are caused by new tools and new cutting patterns, which are charged immediately to the period in which the style is first made, rather than spread over the life of the tool or pattern. However, indirect increases in cost are perhaps more important. They arise from lack of familiarity on the part of foremen

PARTIAL REGRESSION CURVES FOR CORRECTED AVERAGE COMBINED COST WITH SIX INDEPENDENT FACTORS: OUTPUT (X_2), SIZE OF PRODUCTION ORDER (X_3), NUMBER OF NEW STYLES (X_4), INCREASE IN PRODUCTION (X_5), LABOR TURNOVER (X_7), AND QUALITY OF MATERIAL (X_8).

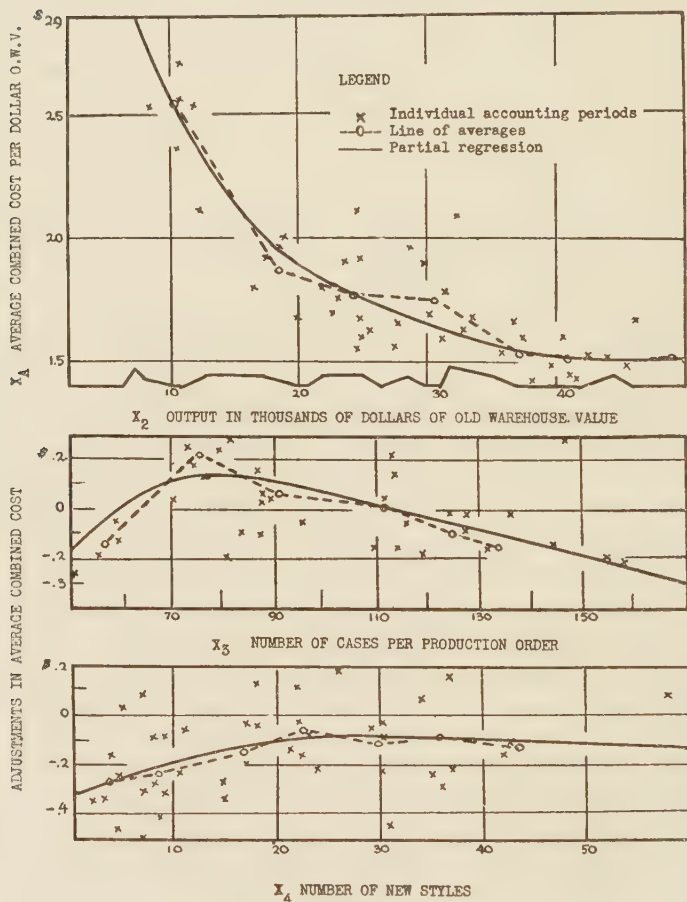


FIG. 2

PARTIAL REGRESSION CURVES FOR CORRECTED AVERAGE COMBINED COST WITH SIX INDEPENDENT FACTORS: OUTPUT (X_1), SIZE OF PRODUCTION ORDER (X_2), NUMBER OF NEW STYLES (X_4), INCREASE IN PRODUCTION (X_5), LABOR TURNOVER (X_7) AND QUALITY OF MATERIAL (X_8).

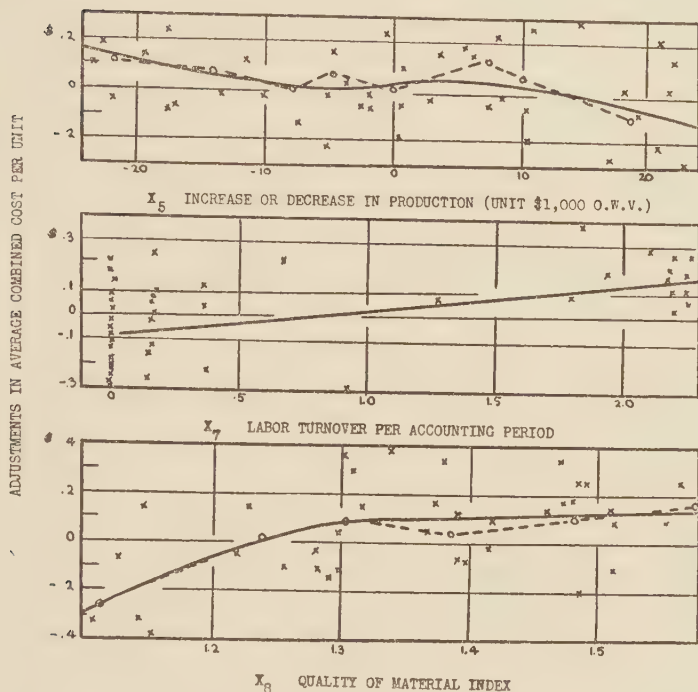


FIG. 2—Continued

and workers with the cutting, assembly, and finishing processes required for the new product.

In the main, the actual course of the combined regression curve for new styles corresponds to its expected behavior: its general direction is upward as the number of new styles increases (Fig. 2). Beyond thirty new styles, however, the curve flattens out, and beyond forty it actually declines very slightly. It may, nevertheless, be true that this phase of the curve is not statistically significant. This condition is suggested by the wide scatter of observations about the regression curve (Fig. 2).

The a priori expectation of the regression curve which was designed to measure the effect of sudden changes in the rate of output from that of the previous period is not entirely clear. Increases and decreases in the rate of production would be expected to produce definite effects upon cost since change itself is expensive and causes readjustments in routines, redivision of labor, and general disorganization.

It is not the amount of the change alone which must be considered, however, but also its direction. Decreases in output would be expected to raise cost per unit, whereas increases might well lower them, for it is more difficult to contract most expenses than it is to expand them.⁷ The curve of cost adjustments for increases and decreases in output would then be expected to have high values for large decreases in output, lower values for smaller decreases, and possibly still lower values for increases in output.

The observed curve (Fig. 2) follows in general this expected behavior. It declines from a high of \$-0.20 for \$24,000 decreases in output to a low of \$-0.12 for \$24,000 increases. However, the decline was not at a constant rate, but was interrupted by a plateau in the vicinity of small increases. No very satisfactory a priori explanation could be found for this interruption. An effort to trace it to some particular element of cost also failed. Consequently, it was deemed probable that this slight aberration from the expected behavior was not significant, but was due to incomplete data rectification, to sampling error, to the use of a cost function which was too flexible, or to the failure to allow adequately by this simple variable for the complicated pattern of change.⁸

⁷ Moreover, because these grudgingly shrinking costs lag behind the rapidly contracting volume, it is probable that a large output contraction would cause a higher cost than a small contraction. Hence it is clear that the curve should have high values for large decreases and lower values for small decreases. Whether the curve should continue downward is less certain. There are, however, two considerations—the limited divisibility of most cost items (as compared with the divisibility of output) and the tendency to overload a productive factor during a brief period of expansion—which make it appear reasonable that the curve of combined cost should continue to decline throughout the range of increases.

⁸ The amount and direction of change in output from that of the previous period represent only part of the pertinent characteristics of this change. The output level from which the increase took place, the number and amount of successive increases

The regression curve for labor turnover rate would be expected to rise as the values of this independent variable increase. Additions to the labor force cause direct expenses of hiring and training, and indirect expenses of low production, spoilage, and general confusion. These expenses are reflected in the index which was chosen to measure turnover rate, since this index is the ratio of the number of new men hired to the average number of men on the force. The regression curve obtained for labor turnover behaved in the manner anticipated (Fig. 2): it rose continuously as the value of the turnover ratio increased.

The regression curve for quality of material would be expected to rise. Clearly, improvement in the quality of materials would logically be associated with higher material, labor, and supervisory costs. Whether or not this increase in cost should occur at a constant rate is not entirely clear. It may be that, beyond a certain point, further increases in the expensiveness of materials do not bring proportional refinements of workmanship. In that event cost would be expected to increase at a declining rather than at a constant rate. The regression curve obtained for quality of materials (X_8) actually does rise at a declining rate in a gentle convex curve (Fig. 2).

In summary, then, the behavior of the net regression curves for corrected combined average cost is largely in harmony with theoretical expectations.

Another method of stating the relationship of average cost to the several independent factors, which is portrayed by the foregoing regression curves, is by means of a tabular exhibit such as Table I. This table shows estimates of cost for various combinations of magnitudes of the independent factors. Such a table may prove to be easier to understand and to use than the regression curves, since estimates of the expected average cost under any given set of operating conditions can easily be made by entering the table at the proper points. However, since the table was prepared from regularly spaced readings from the regression curves, it is merely a more convenient, but slightly less accurate way of presenting these average cost findings.

or decreases preceding this one, the length of time for which the new level could be expected to be maintained, and their combinations and permutations should also be recognized in a complete allowance for the pattern of change in output.

TABLE I*
ESTIMATES OF AVERAGE COMBINED COST UNDER VARIOUS OPERATING CONDITIONS

INCREASE OUTPUT (UNIT OR DE- CREASE OVER PREVIOUS O. W. V.) PERIOD		AVERAGE NUMBER OF PICES PER PRODUCTION ORDER															
		60				80				100				120			
		Number of New Styles				Number of New Styles				Number of New Styles				Number of New Styles			
		10	25	40	10	25	40	10	25	40	10	25	40	10	25	40	10
5.....	-	\$15,000	\$2,864	\$2,966	\$2,959	\$2,971	\$3,076	\$3,066	\$2,904	\$3,009	\$2,793	\$2,808	\$2,888	\$2,687	\$2,792	\$2,782	
	0	2,866	2,911	2,901	2,913	3,018	3,008	2,846	2,951	2,941	2,735	2,840	2,830	2,629	2,734	2,724	
	+	15,000	2,765	2,870	2,860	2,872	2,977	2,967	2,805	2,910	2,900	2,694	2,799	2,789	2,588	2,693	2,683
10.....	-	15,000	2,394	2,499	2,489	2,401	2,506	2,334	2,439	2,520	2,223	2,328	2,418	2,117	2,222	2,312	
	0	2,336	2,441	2,431	2,443	2,538	2,370	2,370	2,591	2,471	2,265	2,480	2,300	2,159	2,374	2,254	
	+	15,000	2,295	2,400	2,390	2,302	2,507	2,497	2,235	2,440	2,124	2,329	2,319	2,018	2,223	2,213	
15.....	-	15,000	1,954	2,059	2,049	2,061	2,166	1,994	2,099	2,089	1,883	1,988	1,978	1,777	1,882	1,872	
	0	1,866	2,001	1,991	2,003	2,108	1,998	1,936	2,041	2,031	1,825	1,930	1,920	1,719	1,825	1,814	
	+	15,000	1,856	1,960	1,950	1,963	2,067	1,896	2,000	1,990	1,785	1,889	1,879	1,679	1,783	1,773	
20.....	-	15,000	1,708	1,813	1,803	1,815	1,920	1,748	1,853	1,843	1,637	1,742	1,732	1,531	1,636	1,626	
	0	1,650	1,755	1,745	1,757	1,862	1,752	1,690	1,795	1,785	1,579	1,684	1,674	1,473	1,578	1,568	
	+	15,000	1,609	1,714	1,704	1,716	1,821	1,649	1,754	1,744	1,538	1,643	1,633	1,432	1,537	1,527	
25.....	-	15,000	1,564	1,669	1,659	1,681	1,776	1,614	1,709	1,699	1,503	1,608	1,588	1,397	1,502	1,482	
	0	1,506	1,611	1,601	1,613	1,718	1,608	1,546	1,651	1,641	1,435	1,540	1,530	1,329	1,434	1,424	
	+	15,000	1,465	1,570	1,560	1,572	1,677	1,595	1,610	1,600	1,394	1,499	1,489	1,388	1,393	1,383	

* As an illustration of a method of presenting results, this fragment of a complete table is shown.

TABLE I—Continued

		AVERAGE NUMBER OF PIECES PER PRODUCTION ORDER															
OUTPUT (UNIT \$1,000 O.W.V.)	INCREASE OR DE- CREASE OVER PREVIOUS PERIOD	60				80				100				120			
		Number of New Styles				Number of New Styles				Number of New Styles				Number of New Styles			
		10	25	40	10	25	40	10	25	40	10	25	40	10	25	40	10
30.....	— \$15,000	\$1.460	\$1.565	\$1.555	\$1.507	\$1.672	\$1.662	\$1.500	\$1.605	\$1.595	\$1.389	\$1.494	\$1.484	\$1.383	\$1.388	\$1.378	
	0	1.518	1.623	1.613	1.625	1.730	1.720	1.558	1.703	1.753	1.447	1.652	1.642	1.341	1.540	1.536	
	+ 15,000	1.477	1.582	1.572	1.584	1.689	1.679	1.517	2.122	1.612	1.406	2.011	1.501	1.300	1.995	1.396	
35.....	— 15,000	1.374	1.479	1.469	1.481	1.586	1.576	1.414	1.519	1.509	1.303	1.408	1.398	1.197	1.392	1.292	
	0	1.316	1.421	1.411	1.423	1.528	1.518	1.356	1.461	1.451	1.245	1.350	1.340	1.139	1.244	1.234	
	+ 15,000	1.275	1.380	1.370	1.382	1.487	1.477	1.315	1.420	1.410	1.204	1.309	1.299	1.098	1.203	1.293	
40.....	— 15,000	1.324	1.429	1.419	1.431	1.536	1.526	1.364	1.469	1.459	1.253	1.358	1.348	1.147	1.252	1.242	
	0	1.266	1.371	1.361	1.373	1.488	1.468	1.306	1.421	1.401	1.185	1.310	1.290	1.079	1.204	1.184	
	+ 15,000	1.225	1.330	1.320	1.332	1.437	1.427	1.365	1.370	1.360	1.254	1.259	1.249	1.148	1.153	1.143	
45.....	— 15,000	1.310	1.415	1.405	1.417	1.522	1.512	1.350	1.455	1.445	1.239	1.344	1.334	1.133	1.238	1.228	
	0	1.252	1.357	1.347	1.359	1.464	1.454	1.292	1.397	1.387	1.181	1.286	1.276	1.075	1.075	1.170	
	+ 15,000	1.211	1.316	1.306	1.318	1.423	1.413	1.251	1.356	1.346	1.302	1.245	1.235	1.196	1.034	1.129	
50.....	— 15,000	1.308	1.413	1.403	1.415	1.520	1.510	1.248	1.453	1.443	1.399	1.342	1.332	1.293	1.031	1.226	
	0	1.250	1.355	1.345	1.357	1.462	1.452	1.290	1.395	1.385	1.341	1.284	1.274	1.235	1.073	1.168	
	+ 15,000	1.209	1.314	1.304	1.316	1.421	1.411	1.249	1.354	1.344	1.300	1.243	1.233	1.194	1.032	1.127	

MEASURES SHOWING THE PROBABLE RELIABILITY OF RESULTS

A second general type of findings resulting from the correlation analysis of average cost behavior consists of measures which show the probable reliability of the average cost curves. Two kinds of reliability measures were computed:⁹ (1) the standard error of the estimate of cost made on the basis of all the regression curves determined for that particular cost item; (2) the standard error of the departures of the output regression curve from the curve reading at the output mean.

If the combined cost per unit for any individual period should be estimated on the basis of all six of the regression curves just described, the estimate would not be exactly correct, even for costs during the three-year period under investigation. A measure of the probable extent of this error is found in the standard error of estimate, which is the standard deviation of differences between actual and estimated cost. If these differences are normally distributed, this standard error of estimate is the range, plus or minus the estimated cost figure, within which the chances are two out of three that the actual cost figure will fall. It should be emphasized that this range is applicable only to accounting periods having fundamental operating conditions similar to those analyzed in this study. The value of the standard error of estimate was found to be \$0.192 (Table II). This is considerably less than the standard deviation of corrected cost (\$0.447), which indicates that considerable improvement in cost estimating resulted from the use of the regression curves.

The second measure of unreliability applies only to the output regression curve. As a provisional indication of the statistical significance of this function, the standard error of departures of the curve from a horizontal line running through the curve value at the output mean (\$1.603) was computed from experimental formulas suggested by Ezekiel.¹⁰ Table III shows these departures and their standard errors

⁹ Error formulas based upon a simple sample theory are not strictly applicable to a selected sample from a time series, especially when these observations appear to be intercorrelated chronologically. Additional error, also not allowed for in these formulas, was added when it was found necessary to drop a few extreme observations from the charts; see chap. v for evaluation of the applicability of these error formulas.

¹⁰ Ezekiel, *Methods of Correlation Analysis*, p. 271.

TABLE II

SUMMARY OF RESULTS OF MULTIPLE CORRELATION ANALYSES

(Comparison of several types of cost with respect to their standard deviation, standard error of estimate, index of multiple correlation, and index of multiple determination)

DEPENDENT VARIABLE	FIGURE SHOWING REGRESSION CURVES	INDEPENDENT VARIABLES† (CODE SYMBOLS)	STANDARD DEVIATION OF DEPENDENT VARIABLE	STANDARD ERROR OF ESTIMATE		INDEX OF MULTIPLE CORRELATION		INDEX OF MULTIPLE DETERMINATION	
				Adjusted for Degrees of Freedom	Unadjusted	Adjusted for Degrees of Freedom	Unadjusted	Adjusted for Degrees of Freedom	Unadjusted
Average combined cost*	2	$X_2, X_3, X_4, X_5, X_7, X_8$	0.447	\$ 0.192	\$ 0.163	\$.906	\$.931	\$.821	\$.867
Average combined cost (uncorrected).....	3	$X_2, X_3, X_4, X_5, X_7, X_8$	0.295	0.230	0.196	.639	.750	.408	.562
Average direct labor cost	9	X_2, X_3, X_4, X_5, X_7	0.2283	0.108	0.095	.867	.878	.752	.771
Average combined indirect cost.....		X_2, X_3	0.0748	0.040	0.038	.986	.987	.972	.974
Average indirect labor cost.....	10	X_2, X_4, X_5	0.0693	0.022	0.020	.949	.958	.901	.918
Average indirect materials cost.....	13	X_2, X_3, X_4, X_5, X_8	0.0072	0.0068	0.0047	.344	.643	.118	.413
Total inspection labor cost.....	11	X_2, X_3, X_4, X_8	87.72	79.59	65.65	.456	.663	.208	.439
Total yard labor cost...	12	X_2, X_3, X_4, X_5, X_9	530.87	392.46	323.82	.689	.793	.475	.628
Total power and watch labor cost.....		X_2, X_3, X_9	138.19	110.82	92.82	.527	.590	.278	.348
Average power cost....	14	X_2, X_3, X_4	0.079	0.062	0.051	.637	.761	.406	.579
Average maintenance cost.....	15	X_2, X_3, X_5	0.076	0.0296	0.0258	.924	.940	.853	.884
Output†.....		X_3, X_4, X_5, X_7	11.62	11.71	9.08	.135	.622	.018	.387

* All costs not otherwise specified have been corrected for price changes and cost-output lags.

† Output was correlated with the other independent variables in order to determine the amount of interdependence among them. The unit is \$1,000 old warehouse value.

‡ X_1 = output; X_2 = size of production order; X_4 = number of new styles; X_5 = change in output; X_7 = labor turnover; X_8 = quality of materials; X_9 = lumber receipts; X_{10} = fuel outlays.

at nine regularly spaced curve points. On the basis of this measure of significance the following conclusions were reached: (1) The deviations of the regression curve from the horizontal are of limited sta-

TABLE III*

MEASURE OF THE STATISTICAL SIGNIFICANCE OF AVERAGE CORRECTED COMBINED COST FUNCTION

(Standard error of difference between partial regression curve at specified value of output and partial regression curve at average value of output)

OUTPUT (UNIT \$1,000 IN O.W.V.)	ESTIMATED COST		STANDARD ERROR OF DEVIATIONS IN COL. 3†
	Reading from Curve	Deviation from Reading at Mean Output†	
(1)	(2)	(3)	(4)
10.....	\$2.469	\$.866	\$.201
15.....	2.038	.405	.246
20.....	1.794	.191	.154
25.....	1.651	.048	.043
30.....	1.548	— .055	.019
35.....	1.458	— .145	.096
40.....	1.409	— .194	.126
45.....	1.397	— .206	.216
50.....	1.397	— .206	.353

* The method of computing this standard error is described in Ezekiel, *Methods of Correlation Analysis*, p. 384. The formulas are provisional and give only a rough approximation.

† Departures of curve readings from the curve value \$1.603, which corresponds to the mean of output (\$29,720).

‡ Standard error refers to departures described in the preceding footnote.

tistical significance in the vicinity of \$45,000 and \$50,000 output. (2) Considerable significance can be attached to these curve departures between \$10,000 and \$40,000 output.

INDEXES SHOWING THE CLOSENESS OF THE RELATIONSHIP BETWEEN
COST AND THOSE INDEPENDENT FACTORS AFFECTING COST

The third general phase of the description of the behavior of average cost is the measurement of the closeness of the relationship between cost and the independent factors. Two kinds of measures of the multiple relationship of all the independent factors to cost were computed for

each cost item: (1) the index of multiple correlation and (2) the index of multiple determination.

The index of multiple correlation measures the closeness of the relationship between cost and the independent factors. It shows the degree of simple correlation between the actual cost magnitudes and the cost values which would be estimated on the basis of the net regression curves. This correlation was found to be high for corrected combined cost. The adjusted value of the index of multiple correlation was computed as .906 (Table II).

The index of multiple determination is the square of the index of multiple correlation. It states the proportion of variance (squared variation) in cost which is "accounted for" by the independent factors. It thus measures the combined importance of the several independent variables as a means of explaining the difference in cost between accounting periods.¹¹ About 82.1 per cent of the variance of combined cost had been accounted for according to the index of multiple determination. The high values of these two correlation constants indicate that most of the influences upon cost have been included in this study.

UNCORRECTED COMBINED COST

In a methodological study it is of interest to know whether the various laborious refinements in procedure make any difference in the results. Did correcting the cost data and allowing for independent factors by means of multiple correlation result in different and more sensible average cost findings than simpler methods would have given? To answer this question two special analyses of uncorrected combined cost were made: (1) a simple visual regression analysis of the gross relationship of uncorrected combined costs with output and (2) a complete graphic multiple correlation analysis of uncorrected cost with the same independent variables which were employed for corrected combined cost.

The first of these special analyses obtained an output regression curve for average cost by visually fitting a gross rather than a partial regression curve to the uncorrected cost data. A comparison of this average cost curve with the partial regression curve for corrected cost

¹¹ *Ibid.*, p. 178.

STATISTICAL DETERMINATION OF COSTS

PARTIAL REGRESSION CURVES FOR CORRECTED AVERAGE COMBINED COST COMPARED WITH PARTIAL REGRESSIONS FOR COST NOT CORRECTED FOR PRICE CHANGE AND OUTPUT LAG AND WITH GROSS REGRESSION OF WHOLLY UNCORRECTED COST -- PARTIAL REGRESSIONS FOR X_2 , X_3 , X_4 , X_5 , X_7 , X_8 ; GROSS REGRESSION FOR X_2 .

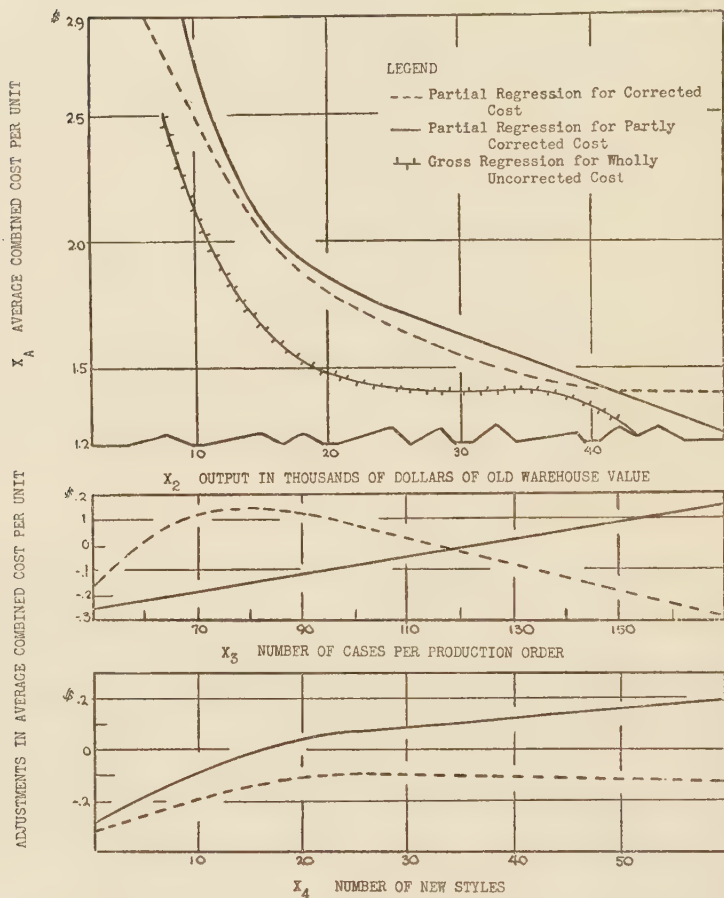


FIG. 3

PARTIAL REGRESSION CURVES FOR CORRECTED AVERAGE COMBINED COST COMPARED WITH PARTIAL REGRESSIONS FOR COST NOT CORRECTED FOR PRICE CHANGE AND OUTPUT LAG AND WITH GROSS REGRESSION OF WHOLLY UNCORRECTED COST -- PARTIAL REGRESSIONS FOR X_2 , X_3 , X_4 , X_5 , X_7 , X_8 ; GROSS REGRESSION FOR X_2 .

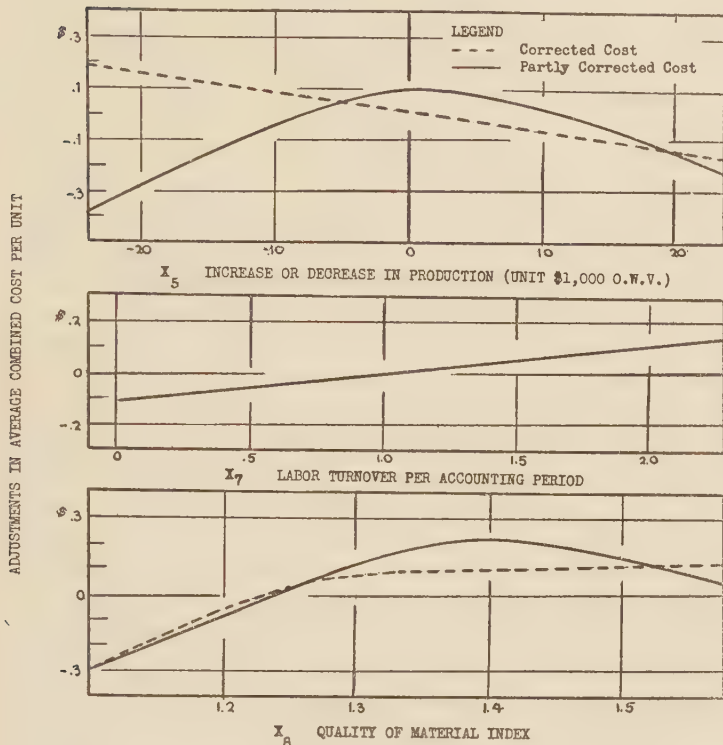


FIG. 3—Continued

(Fig. 3) reveals several important differences. The former curve falls more steeply, flattens earlier, and then renews its fall more precipitously when large output levels are reached. Since these departures do not appear reasonable, they indicate that either data correction or multiple correlation analysis, or both, are essential steps in obtaining satisfactory average cost results.

The second subsidiary analysis of uncorrected cost was designed to find out whether both these steps in the preparatory analysis were necessary. A complete multiple correlation analysis identical to that

made for corrected cost was made with data which were not corrected for rate changes or for output lag. The partial regression curves obtained by this analysis are contrasted with those for corrected cost in Figure 3. Differences in the results of these two analyses were obviously ascribable solely to data corrections. Differences between the net output regression curve for uncorrected cost and the gross regression for uncorrected cost, shown in Figure 3, on the other hand, are clearly attributable to the effects of multiple correlation analysis.

A careful comparison of these three sets of curves and a comparison of the corresponding correlation constants (Table II) led to the following tentative conclusions regarding the justification for the foregoing two steps in data rectification, as indicated by their effect upon the behavior of average cost in the particular firm studied:¹²

1. Multiple correlation analysis appeared to be necessary in order to stabilize the effects of other cost influences and thus secure a sensible regression curve for average cost and rate of output.

2. The correction of cost data for price change and for output lag seemed to be a necessary preliminary step for the determination of satisfactory curves for several of the independent factors. Rectification of the cost data apparently brought the curves more nearly into accord, in several important respects, with reasonable a priori expectations. In the first place, the output regression curve (X_2) for corrected data, instead of declining continuously, flattened toward the horizontal in the region of high production activity. Second, the slope of the regression for size of the production order (X_3) was reversed and thus made reasonable. Third, the unreasonable slope of that portion of the regression curve for changes in output (X_5), which applies to decreases, was reversed in the corrected curve; thus the entire curve was made to accord more nearly with expectations. Fourth, the illogical downturn of the uncorrected regression for quality of material (X_8) was removed, leaving a corrected curve which rose reasonably over its entire course.

3. The accuracy with which costs can be estimated on the basis of the values of the independent factors was increased by rectifying the

¹² The peculiarities of cost behavior of a particular firm are not a very sound foundation for generalizations regarding the validity of a method. It must be recognized that such correction is necessary only when the distortion removed would affect results to a quantitatively significant degree.

cost data. This is shown by the reduction of the adjusted standard error of estimate from \$.230 for uncorrected cost to \$.192 for corrected cost.

4. The degree of success with which cost behavior was accounted for by the independent variable was enhanced by rectifying the cost data. The adjusted index of multiple correlation rose from .639 for uncorrected cost to .906 for corrected cost, and the adjusted index of multiple determination rose from .408 to .826.

Apparently, clearing away the effects of irrelevant influences, such as price change and time lag, brought out more definitely the true relationship between cost and the various independent factors. The practical importance of a reasonably accurate determination of these crucial relationships is clear. If these curves are to be the basis for determining adjustable standards for cost control, it is important that these adjustments be correct. Otherwise the standards will not command respect. Similarly, if the curves are to be used as aids in budgeting expenses (and profits), rectification of data which will greatly improve the accuracy of estimates is highly desirable.

CHAPTER VII

BEHAVIOR OF MARGINAL COMBINED COST

The present chapter deals with the findings for marginal combined cost.¹ These findings are of interest to management because estimated future marginal cost, considered in connection with anticipated marginal revenue, is necessary in determining the selling price and the output level which will yield the maximum profit. These findings are also of interest to economists since the behavior of the marginal cost function has been the subject of much speculation on their part but has, notwithstanding, elicited little statistical investigation. And, finally, these results may interest the statistician because they throw some light upon various alternative procedures for obtaining estimates of marginal cost.

SUMMARY OF CONCLUSIONS

Extensive experiments with several methods of deriving marginal cost from corrected data in the form of averages and in the form of totals lead to the following tentative conclusions.

1. More satisfactory estimates of marginal cost can be obtained from data in the form of totals than from data in the form of averages.
2. In working with total cost data least-squares curve fitting gives satisfactory results when the proper function is chosen. Visual fitting is more economical, however, and apparently almost as reliable particularly when, as in this case, the total cost function is linear.
3. The most reliable estimate of marginal cost which can be made from these data is that it is constant at \$1.12 per-dollar increase in old warehouse value of output regardless of the rate of output. Curvilinear functions were also fitted which yielded varying marginal costs,

¹ By marginal cost is meant the increment in cost which is consequent upon a unit increment in output. In this study that unit is one dollar of old warehouse value. The curve of marginal cost is defined as the locus of this increment in cost over a given range of output. Cost increments associated with other operating conditions are also of interest to management. They are discussed at the end of the chapter.

but they are not particularly reasonable and they squeeze more out of the data than their limited reliability justifies.

4. In dealing with cost data in the form of averages none of the methods employed gives very reliable marginal cost estimates. Of the procedures tried, the graphic method proved most satisfactory and the method of first differences least trustworthy. Although both the derivative (least-squares) and the graphic-derivative procedures demonstrated important advantages, they proved to have weaknesses which were particularly dangerous in dealing with variable data in average cost form.

5. The adequacy and reliability of the best estimate of marginal cost (\$1.12) are limited by several considerations: (a) this estimate represents only marginal manufacturing costs; (b) correction of the data is not perfect; (c) the sample may not be representative of subsequent periods; and (d) measures of the reliability of average and total cost functions which are based upon the dispersion of the data and the size of the sample exaggerate somewhat the degree of confidence which can be placed in these functions and also the reliability of the marginal cost estimates derived from them.

Experimentation with data representing several stages of correction led to the general conclusion that full rectification of the data was necessary in order to obtain reliable estimates of marginal cost. This conclusion was based upon the following findings:

1. Estimates of constant marginal cost made from incompletely corrected data were different from those obtained from corrected data.

2. These estimates were also less reliable since the dispersion of observations was greater.

3. Correction of the data brought significant changes in the shape of the marginal cost function.

4. Partial correction of the data for price change and time lag alone did not suffice when, as in this case, there were other important operating conditions which affected cost behavior.

5. Partial correction by multiple correlation analyses alone did not give reliable marginal cost estimates, at least not during periods of pronounced price and volume variability.

6. A short-cut procedure yielding quick, rough estimates of constant marginal cost by visually fitting a straight line to the raw total cost

data might prove usable under conditions of greater price and volume stability and when other cost factors did not cause important distortion of the cost-output relationship.

MARGINAL COST ESTIMATED FROM CORRECTED DATA BY
ALTERNATIVE METHODS

The first phase of the experiment involved a test of several alternative procedures for deriving marginal cost from fully corrected combined cost data. These procedures may be classified in two groups according to the form of cost data used. The first group employed data in the form of the adjusted total cost for each accounting period, whereas the second used data in the form of the adjusted average cost per unit of output for the period. To the first type of data two kinds of function were fitted both by the method of least squares and by visual methods: (a) a straight line and (b) a second-degree parabola. To the second kind of data, curves of various types were fitted, and marginal cost was derived by several fairly distinct procedures. It will be noted that these procedures differ not so much according to the type of function fitted, as according to the method of fitting the function and of deriving marginal cost from it. The methods used in studying these average cost data may be called:

1. The graphic method (visual curve fitting)
2. The first-differences method (no curve fitted)
3. The derivative method (least-squares curve fitting)
4. The graphic-derivative method (selected-points curve fitting)

The relative merits of the total as against the average cost methods should be noted at this point. Total cost data proved more reliable because marginal cost is a simple function of the total cost curve (i.e., its slope). Hence, in fitting the total cost function with this in mind, its slope can be prevented from having unwarranted fluctuations. On the other hand, marginal cost is a complicated function of the average cost curve. Hence, in fitting this curve, spurious fluctuations of its marginal cost function cannot be easily controlled. If, however, comparable functions had been fitted both to totals and to averages, for example:

$$X_T = a + bX_2 \text{ (for totals)}$$

$$X_A = \frac{X_T}{X_2} = \frac{a}{X_2} + b \text{ (for averages) ,}$$

then results of about equal reliability could have been obtained from the average cost data.

Since parallel functions of this type were not employed (they would have been difficult to hit upon by graphic methods), observations of total cost were found to be superior to observations of average cost as a basis for deriving estimates of marginal cost. Two chief advantages resulted from using data in this form. First, marginal cost curves appeared to be somewhat more reliable and more reasonable since they were freed from the spurious fluctuations given them by amplification of faint erroneous changes of slope in the average cost curves. Second, marginal cost curves could be secured more economically since, when the entire analysis was carried out in terms of totals, two steps were obviated: (1) conversion of individual values into per-unit observations and (2) subsequent transformation of the average cost function into a total cost function.²

METHODS EMPLOYED FOR TOTAL COST DATA

The best estimate of marginal cost which could be made from the available data was that it is constant at about \$1.12 per dollar of old warehouse value (see Table IV). This figure was arrived at by fitting a straight line by the method of least squares to fully rectified cost data in the form of totals and taking the first derivative of the resulting total cost equation.

The least-squares method was chosen for fitting a curve to these preferred total cost data because it is more objective than visual fitting and because marginal cost cannot only be derived precisely from the resulting total cost equation but also, can be expressed concisely in the form of an equation. But a line fitted by visual methods would have given approximately the same results in a fraction of the time. Graphic methods were used satisfactorily in deriving marginal cost from uncorrected combined total cost data and from the total cost observations of its components.

² The procedure actually followed for corrected combined cost totals differed from that for data in the form of averages primarily in the step at which the conversion from averages to totals took place. With average cost data the transformation came after a function had been fitted to the observations; with total cost data, on the other hand, the conversion was affected individually for each accounting period before fitting a curve.

A straight line was chosen as the preferred function to be fitted by least squares to these total cost data. In the first place, the distribu-

TABLE IV*
COMPARISON OF MARGINAL COMBINED COST OBTAINED BY VARIOUS
METHODS FROM FULLY CORRECTED COST DATA IN THE FORM OF
AVERAGES AND IN THE FORM OF TOTALS

OUTPUT (UNIT \$1,000 O.W.V.)	DERIVED FROM AVERAGE COST DATA†			DERIVED FROM TOTAL COST DATA‡	
	First- Differences Method§	Derivative Method	Graphic Method¶	Least- Squares Straight Line Function	Least- Squares Parabolic Function
10.....	1.52	1.52	1.62	1.12	1.54
15.....	1.02	1.06	1.17	1.12	1.43
20.....	1.01	0.98	1.18	1.12	1.31
25.....	1.14	1.06	1.17	1.12	1.20
30.....	1.50	1.21	1.07	1.12	1.08
35.....	0.10	1.30	1.04	1.12	0.96
40.....	1.25	1.15	1.27	1.12	0.85
45.....	1.43	0.70	1.41	1.12	0.73

* Data used in each of these procedures were adjusted by price index and time lag correction and by multiple correlation analysis.

† Derived by fitting functions to average cost residuals, then transforming these functions into terms of total cost and taking the first derivative of the latter function.

‡ Derived by first transforming individual average cost residuals into total cost residuals then fitting functions to these observations.

§ Tabular-graphic derivation from unsmoothed group averages of per-unit cost residuals (Fig. 5).

|| Mathematical derivation from cubic equation fitted to average cost residuals (Fig. 5).

¶ Tabular-graphic derivation from smoothed curve fitted visually to average cost residuals (Fig. 5).

tion of these corrected data appeared to be approximately linear (Fig. 4). In the second place, the data, even in this refined form, were thought to be too imperfect to justify representation by complex functions. A second-degree parabola,³ which was fitted to these data by

³ This parabola was found to have the following equation:

$$X_T = 5.46 + 1.7759X_2 - .116X_2^2.$$

When differentiated, this equation yielded as the marginal cost function the following declining straight line:

$$X_M = 1.7759 - .0232X_2.$$

ESTIMATING MARGINAL COMBINED COST FROM ADJUSTED TOTAL COST OBSERVATIONS BY
FITTING A STRAIGHT LINE AND A PARABOLA BY THE METHOD OF LEAST SQUARES

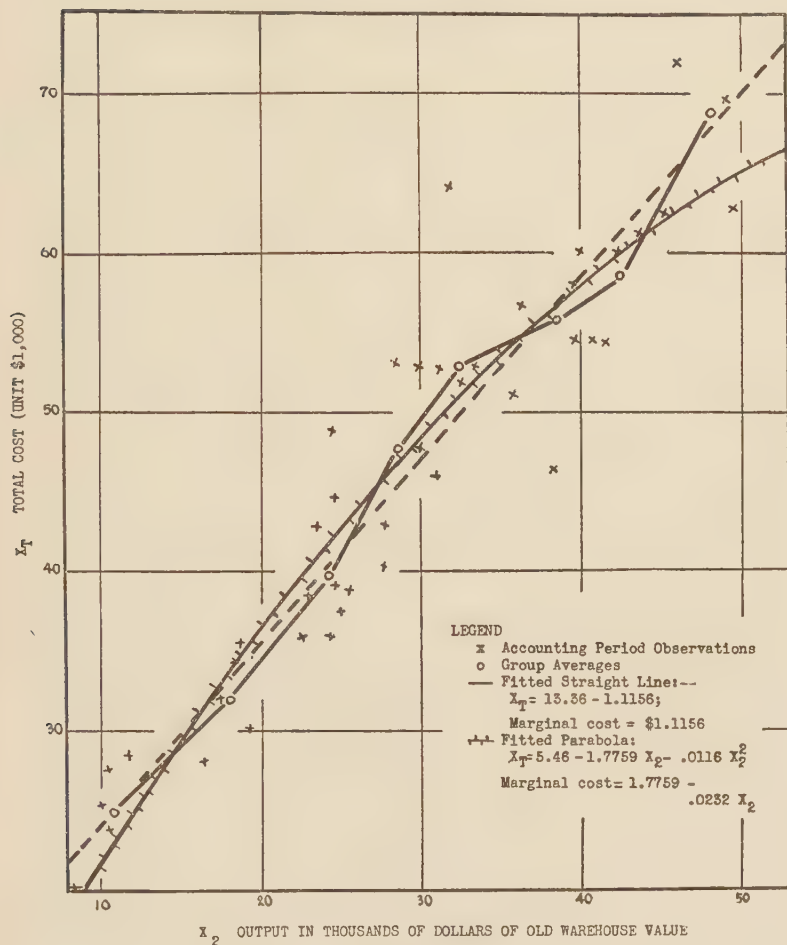


FIG. 4

least-squares procedures, did not represent the data well at large outputs, as can be seen from Figure 4. From Table IV, where the first derivative of this parabolic function is evaluated, it is clear that this estimate indicates that marginal cost declines continuously as output increases.

In the short period both constant marginal cost and declining marginal cost go counter to ordinary theoretical expectations since such behavior is generally not considered compatible with individual equilibrium for firms operating under conditions of effective competition.⁴ Under conditions of monopolistic competition, on the other hand, with the consequent tendency toward continuous overcapacity, with a sharply rising selling cost curve and a resulting declining net individual demand schedule, and with considerable friction, constant or declining marginal manufacturing costs over the range of practical operation are compatible with equilibrium for the individual firm.⁵

METHODS EMPLOYED FOR AVERAGE COST DATA

The second group of methods used data in the form of average cost per dollar of old warehouse value for each accounting period. Only by trying data of this second type and comparing the results with those obtained from total cost data could the superiority of the total cost procedures be established. Such a comparison of results is facilitated by Table IV which contrasts the estimates of marginal cost derived from total cost data with those obtained by four methods from average cost data. Figure 5 shows the same information in graphic form. Of the various procedures for deriving marginal cost from average cost

⁴ Jacob Viner makes the suggestion that neoclassical theory may not have taken enough factors into consideration. The quasi-physical relationships considered in the law of diminishing returns may be more than offset by organizational and psychological influences which work counter to them.

⁵ There is some question whether marginal production costs are more likely to be constant or declining under monopolistic competition than under perfect competition. Much depends upon the conditions of supply of the input factors. The point at issue here is the conditions for individual firm equilibrium. Although equilibrium for the economy as a whole is never realized and even though there may not even be a tendency toward such equilibrium, equilibrium for the individual firm does tend to be approached if the enterprise is rationally managed.

COMPARISON OF ESTIMATES OF MARGINAL COMBINED COST OBTAINED FROM CORRECTED DATA BY SEVERAL ALTERNATIVE METHODS USING DATA IN THE FORM BOTH OF AVERAGES AND OF TOTALS

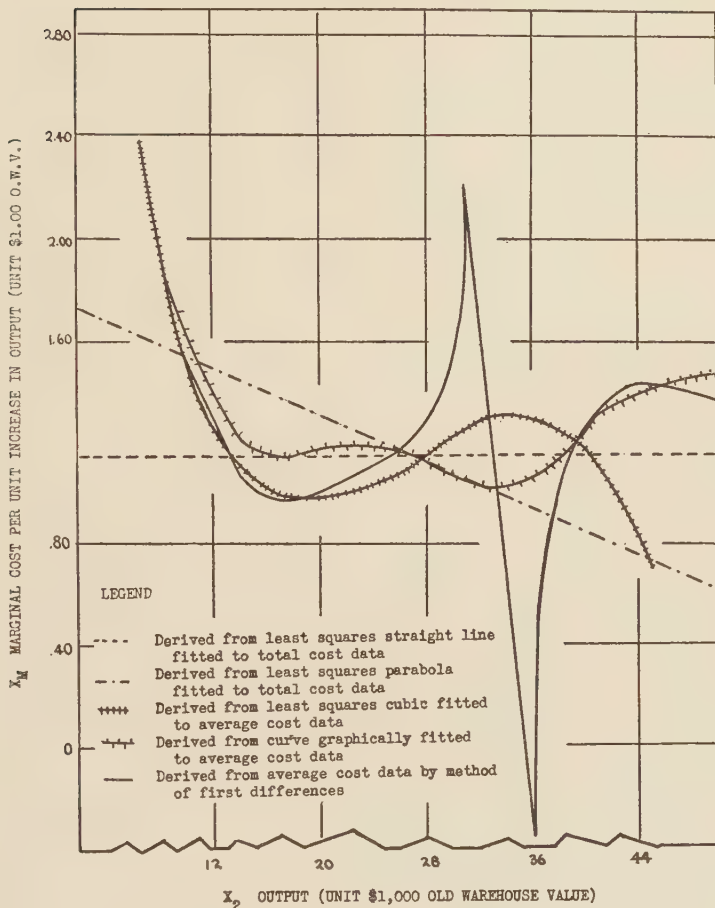


FIG. 5

data the graphic method proved to be most satisfactory.⁶ The incremental cost curve obtained by this method is shown in Figure 5.

From the fact that the two striking peculiarities of this curve—its brief initial declining phase and its upturn at the end for high rates of production—appear reasonable under assumptions of effective competition, it might seem that the graphic procedure is to be preferred over other methods of deriving marginal cost from average cost functions. However, it must be borne in mind that the assumptions of atomistic competition probably do not hold for the present firm's manufacturing cost. It has been noted that, under conditions of imperfect competition, with rising marginal selling costs and declining individual demand schedules, marginal manufacturing cost may not rise at all over the relevant range of output. Hence, the shape of the marginal curve obtained in this case cannot be advanced as an argument for the use of the graphic method in other investigations, particularly in view of the fact that the fluctuations of this curve do not appear to be statistically significant (judging from the linearity of the distribution of the total cost residuals in Fig. 4).

It is for sounder reasons that the graphic method is preferred for commercial derivations of marginal cost from average cost data. (These advantages apply equally to total cost data.) First, visual curve fitting is flexible since choice of the proper function and evaluation of its parameters is simplified; second, it is able to abstract from non-typical variations in the data; third, the method is economical, particularly when graphic multiple correlation analysis is a step in the rectification of the cost data; and, fourth, the procedure and its results are easily understood.

The method of first differences gave less satisfactory results than the graphic procedure. The marginal cost curve secured from fully rectified data by this method is depicted in Figure 5. Fluctuations of this curve are so violent and so irrational that they are of no managerial interest. Marginal cost estimates secured by its use were unreliable, were stated with little precision, and were determined at no saving in labor.

⁶ This procedure consists of: (1) visual fitting of a curve to the average cost observations; (2) tabular transformation of this curve into a total cost series; and (3) arithmetic differentiation of this function into a marginal cost series of first differences which is smoothed, by means of moving averages, to a marginal cost curve.

When a cubic of the progressive series was fitted to the average cost data by the method of least squares and the resulting equation converted into a total cost function and differentiated, a different sort of marginal cost curve was obtained. This curve is compared in Table IV and in Figure 5 with estimates computed by other methods. The most important departures of this curve from the estimate of constant marginal cost are: (1) it has high values for small outputs and declines sharply as output increases in this early range; (2) it rises gently in the middle range of outputs; and (3) it falls continuously for large outputs (Fig. 5).

The peculiarities in the behavior of this curve are largely attributable (a) to the use of data in the form of averages rather than in the form of totals, (b) to the type of function chosen, and (c) to the method employed in fitting it to the data. Because the data were of inferior form, because the function chosen was not the proper type,⁷ and because the method of least squares does not abstract from what appear to be non-typical data, the marginal curve cannot be considered reliable. It is merely another example of what not to do.⁸

The graphic-derivative method is a hybrid technique which attempts to preserve some of the advantages of the two procedures which it combines. From the graphic method it possesses advantages in curve fitting in the form of economy, flexibility, and freedom from sampling errors; and from the derivative method, precision and conciseness in differentiating and expressing functions. However, these advantages

⁷ As noted above, a hyperbolic function plus a constant would have paralleled the total cost function which gave reliable results.

⁸ The least-squares-derivative method has, however, some important superiorities over other techniques of deriving marginal cost from average cost series. In the first place, its least-squares method of curve fitting yields an average cost curve which is definite and comparatively free from personal bias. In the second place, the marginal and the total cost curves are related to average cost in a precise manner which assures the internal consistency of the findings. And, in the third place, the results of the cost analysis are expressed in a form which, while concise, is yet sufficiently general to determine the curve precisely for all values of output. However, to offset these advantages of objectivity, precision, and conciseness we find certain defects: (1) the curve may fail to represent actual cost behavior because of poor choice of function or inability to abstract from non-typical cost behavior; (2) the procedure is laborious and somewhat difficult to understand; and (3) the results are difficult to interpret to the group who will use them.

are offset in part by three defects which are, in some cases, important: (1) location of the curve by visual methods is somewhat subjective and arbitrary; (2) there is a possibility that the average cost equation will not accurately represent the average cost curve; and (3) it is certain that added labor will be required for the two-way transformation of graphic results into terms of mathematical symbols, then back again into terms of tables or graphs. Estimates of partially corrected combined marginal cost obtained by this method are shown in Table V. Estimates of components of combined cost, derived by this same procedure, are found in Appendix B. This method is not considered trustworthy when average cost data are used.

In closing, it should be emphasized that it is not recommended that marginal cost be derived, by any of these methods, from data in the form of averages.

RELIABILITY OF MARGINAL COST FINDINGS

The best estimate of marginal cost which could be made from the fully corrected data was a constant, \$1.12. This estimate, although more trustworthy than estimates computed by other methods and those made from less refined data, is still subject to several limitations which restrict its reliability. (1) It represents only marginal manufacturing cost. To this must be added an estimate of marginal selling and administrative cost before a figure can be arrived at which will be serviceable for price and output policy.⁹ (2) There are some doubts about the completeness of the rectification of the data. (3) There is some question about the representative character of the sample. Eagerness to select a period full of challenging methodological problems may have resulted in findings which are less typical of subsequent periods than those which could have been secured from data of a more stable period than the depression years of 1932, 1933, and 1934.¹⁰

⁹ The omission of cost items whose total is unchanged by changes in output in no way influences marginal cost, for the derivative of a constant is zero. Some administrative and selling items are of this type.

¹⁰ Quite apart from changes in the prices of input factors (for which allowance can easily be made), the character of the cost relationship itself may be different for various phases of the business cycle. The marginal cost function obtained here is based upon the assumption that the combination of components found at each level of output is the least cost or optimum combination for that output. Cyclical forces may cause fluctuation in the degree to which this assumption holds true.

And (4) the degree of reliability of this estimate, which is measured on the basis of the size of the sample and the variability of the data, cannot be very satisfactorily determined.¹¹

The considerations mentioned above must be kept in mind in using this estimate (\$1.12). They should not, however, lead us away from the important truth that this computed marginal cost is reasonably reliable, especially for the middle portion of the output range (fundamental cost conditions remaining the same). The data are essentially accurate, though incomplete, and the rectification was sufficiently satisfactory for obtaining serviceable marginal cost estimates.

ALTERNATIVE DEGREES OF DATA REFINEMENT

In addition to testing alternative procedures for deriving cost from corrected data a second general experiment in methodology was performed in order to determine whether or not the correction of these data was necessary. This is a very practical question since the expense of rectifying data by means of specially computed index numbers, lag allowances, and multiple correlation analyses is great. If equally satisfactory marginal cost estimates can be obtained without this modification of the cost data, the saving in research costs will be great.

Some light has been thrown upon this problem in the course of the planning of the correction procedures, but not enough, however, to constitute convincing evidence that the correction procedure was necessary. Errors which were removed by correction may have been equally distributed with respect to the relationships under study. In that event their distorting effect would have been negligible.¹² The significant

¹¹ One previously discussed measure of statistical significance is the standard error of departures of a net regression curve from the curve value at the mean of output (Table III). Another measure of reliability is the standard error of estimate about the functions fitted to total cost residuals. For the straight line this constant was found to be \$4,243 (O.W.V.) and for the second-degree parabola, \$3,763 (O.W.V.). However, the width of these bands exaggerates the reliability of these functions since the departures of observations from these curves are actually serially correlated. A more pertinent measure of the reliability of this marginal cost estimate would be the standard error of the slope of the total cost net regression for output. Unfortunately such standard errors cannot yet be determined for graphic correlations.

¹² W. L. Crum, in his statistical study of railroad costs, indicated his belief that the distortion of his cost data was distributed in such a manner as to have little

test of the necessity for the rectification procedure lies not in its effects upon the data, but in its effects upon the marginal cost results. Previous investigation of the effect of correction upon the observed curves of average cost indicated that nothing short of complete rectification of the data would give satisfactory knowledge of average cost behavior. It might be held that this is sufficient evidence of its effect upon marginal cost since, if the average cost curves are not reliable, marginal cost functions derived from them will be even more distorted. It will be of interest, however, to perform a parallel experiment by computing marginal cost estimates from total as well as from average cost data representing successive stages in the process of rectification in order to compare these results with those obtained from fully corrected data.

Three steps or levels of data correction were represented in this experiment. On the lowest level stand wholly uncorrected cost observations.¹³ On the second level of refinement stand data which have been corrected for changes in prices and for discrepancies in the time at which price and output were recorded, but which have not been adjusted (by means of a multiple correlation analysis) for other influences which affect production cost and distort the relationship between cost and output. The third stage is represented by data which were not corrected for price fluctuations and output lag, but which were rectified by means of a multiple correlation analysis.

Data representing these various stages were studied in the form of totals and then in the form of averages in order to check the conclusions regarding this important practical question. Comparison of the marginal cost functions derived from the incompletely corrected data in both the average and the total form, with the best results obtained from corresponding corrected data, led to the general conclusion that full rectification of the data was necessary in order to secure reliable marginal cost results.

effect upon the relationship under investigation ("Statistical Allocation of Joint Costs of Railroad Freight and Passenger Service," *Journal of the American Statistical Association*, XXI, 22).

¹³ These data had been adjusted only by the omission of three unimportant accounts (teaming, traveling, and interplant prorations). These omissions were necessary in order to make the findings comparable since these accounts had also been dropped from the refined cost data.

EXPERIMENTS WITH UNCORRECTED DATA IN THE FORM OF TOTALS

On the basis of experiments with data in the form of totals several conclusions regarding data correction may be drawn. First, estimates of constant marginal cost made by fitting a straight line to these uncorrected total cost observations were different from those obtained from corrected data. For wholly uncorrected data the estimate was \$1.22 per dollar of old warehouse value; for price and lag corrected data it was \$1.09; for data corrected by multiple correlation analysis alone marginal cost was \$0.79; and for fully corrected data it was \$1.12. This disparity throws doubt upon estimates based upon other than fully corrected data. Second, the dispersion of observations was materially reduced by rectification, as can readily be seen from a comparison of Figures 4, 6, and 8. This indicates that the confidence which can be placed in the estimates of marginal cost was considerably increased by correcting the data.

Third, in addition to reducing the dispersion, correction of the data brought the form of the total cost distribution nearer to linearity. Examination of Figures 4, 6, and 8 shows that each of these distributions could be better represented by a curve than by a straight line; on the other hand, the distribution of corrected cost in Figure 4 is approximately linear. This effect of rectification buttresses the case for constant marginal cost. From Figure 6 it can be seen that wholly uncorrected cost data would yield estimates of declining rather than of constant marginal cost and that this erroneous estimate would be poorly supported by the widely scattered data. Figure 7 shows that, even after correcting the data for price change and output lag, the distribution of total cost observations remained definitely curvilinear and indicated a declining, rather than a constant, marginal cost.¹⁴ From Figure 8 it is clear that data corrected by multiple correlation analysis alone yielded unsatisfactory marginal cost estimates. A convex parabola is clearly called for by the distribution of total cost

¹⁴ It was suggested in chap. iii that the prices of input factors may be positively correlated with output. If this correlation is quantitatively significant and if this type of price variation has been removed by our correction procedure (which is doubtful), then corrected marginal cost would be more likely to decline than would uncorrected. The fact that the reverse was found to be true indicates that this type of price variation was not important but that the price and lag corrections, taken together, were actually negatively correlated with output.

MARGINAL COMBINED COST ESTIMATED FROM WHOLLY UNCORRECTED TOTAL COST
OBSERVATIONS

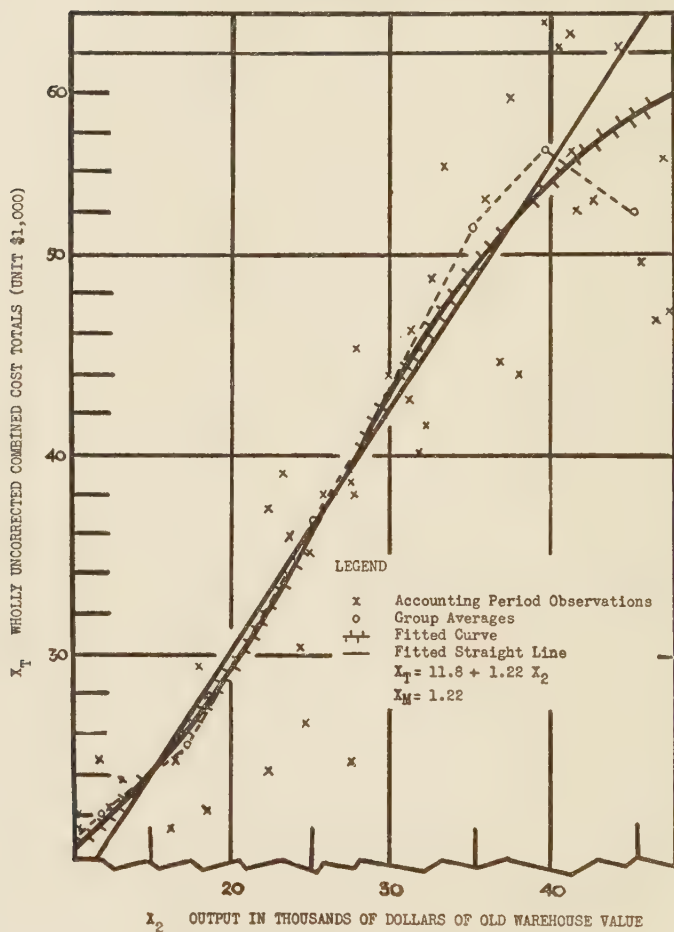


FIG. 6

MARGINAL COMBINED COST ESTIMATED FROM TOTAL COST OBSERVATIONS CORRECTED
FOR PRICE CHANGE AND OUTPUT LAG ONLY

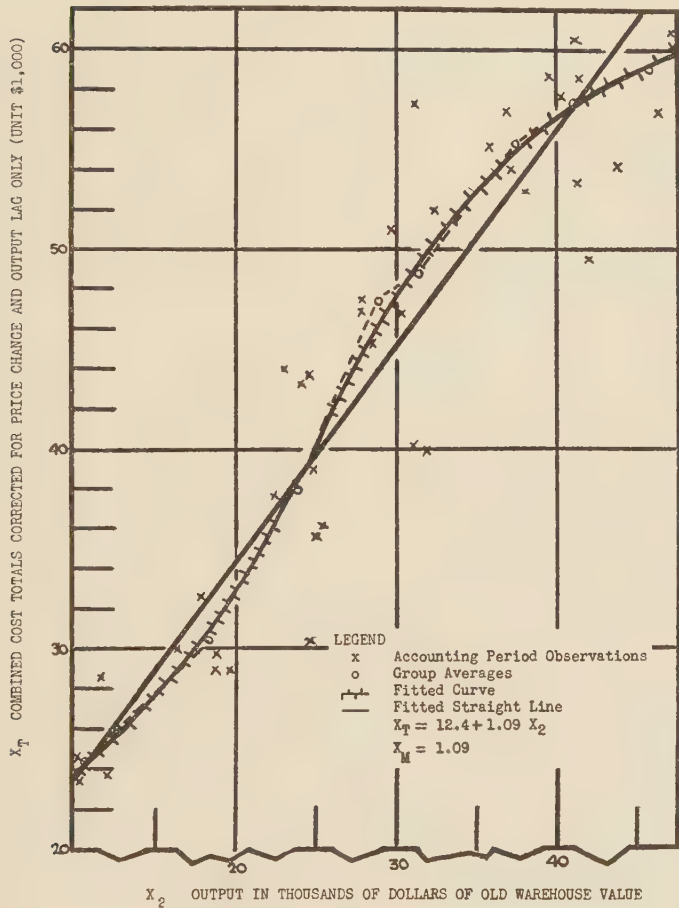


FIG. 7

observations. This type of total cost function will yield a continuously declining straight line as the estimate of marginal cost.

It is clear, from what has been said, that partial rectification of the data by means of price and time-lag correction alone, or by means of multiple correlation analysis alone, will not suffice during periods of great price and volume variability. Correction for changes in prices and wage rates may not, however, be necessary in studies covering periods of greater price stability. Correction for the lag of output behind recorded cost, on the other hand, although particularly needed in this period of output instability, would probably have to be retained even during a more stable period for those firms in which this type of disparity existed. Correction by means of multiple correlation analysis could be dispensed with in similar studies only if the effect of other variables upon cost was either unimportant or randomly distributed. The distribution of Figure 7 holds out considerable hope for this sort of short cut.¹⁵

EXPERIMENTS WITH UNCORRECTED DATA IN THE FORM OF AVERAGES

The general conclusions drawn from experiments with data in the form of totals were substantiated when the effects of the correction process were tested with data in the form of averages. The principal additional contribution of this latter experiment was to show explicitly the changes in the shape of the marginal cost function brought about by correction. These alterations are shown in Table V, where marginal cost curves derived from wholly uncorrected average cost data and from data corrected by multiple correlation analysis alone (called partially corrected) are contrasted with the curve derived by the same average cost method from completely corrected data. Examination of Table V shows that agreement among these curves is confined to their initial falling phase; beyond \$15,000 output the disparity is pro-

¹⁵ The possibility of an abridged procedure which would effect enormous savings is suggested by the improvement in accuracy and reliability resulting from this latter partial correction. Under certain conditions even this correction could be dispensed with and rough approximations to marginal cost could be made by visually fitting a function to the raw total cost data. This procedure could be used, however, only during periods of relative price stability when no important output lag existed and when factors other than rate of production did not cause important cost distortion.

MARGINAL COMBINED COST ESTIMATED FROM TOTAL COST OBSERVATIONS CORRECTED
BY MULTIPLE CORRELATION ANALYSIS ONLY

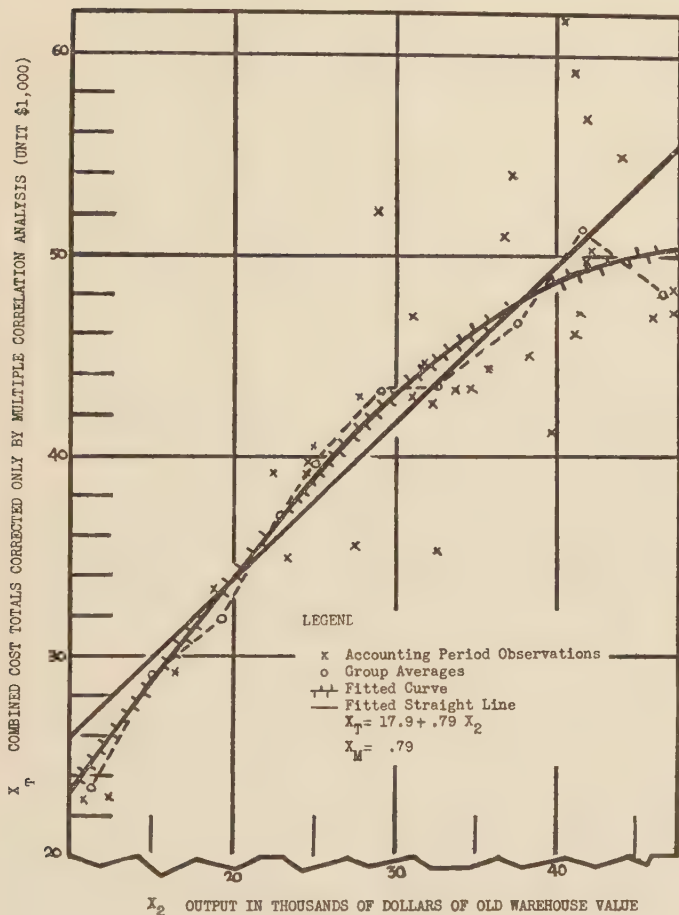


FIG. 8

nounced. Evidently, correction of the data substantially altered the shape of the marginal cost function.

TABLE V
COMPARISON OF ESTIMATED MARGINAL COSTS FROM
CRUDE, FROM PARTIALLY CORRECTED, AND
FROM REFINED AVERAGE COST DATA

OUTPUT (UNIT \$1,000 IN O.W.V.)	MARGINAL COST ESTIMATES		
	From Crude Data*	From Partially Corrected Data†	From Re- fined Data‡
10.....	\$0.94	\$1.50	\$1.50
15.....	0.72	1.14	1.07
20.....	1.02	0.97	1.09
25.....	1.27	0.91	1.07
30.....	1.39	0.86	0.98
35.....	1.30	0.75	0.93
40.....	0.55	0.48	1.07
45.....	— 0.20	— 0.03	1.33
50.....		— 0.87	1.38

* Data were not corrected in any way; and marginal cost was derived by the graphic method.

† Data were adjusted by multiple correlation analysis but were not corrected for price change or for output lag; marginal cost was derived by the graphic-derivative method.

‡ Data were adjusted by correlation analysis and by rate and time lag correction; marginal cost was derived by the graphic method.

OTHER MARGINS OF COMBINED COST IMPORTANCE

Up to this point in the analysis of marginal cost we have devoted attention exclusively to the output margin of cost. But from our multiple correlation analyses we know that cost is really a function of many operating conditions of which the rate of output is but one. Consequently, cost has other margins beside the output margin. For example, increasing the number of new styles has an incremental cost which is analogous to that resulting from increase in output. Moreover, each of these independent variables may itself have several dimensions. Output, for example, often has an important quality margin as well as quantity margin.¹⁶

¹⁶ The finance company whose data are used for the analysis of long-period cost behavior attempted to measure and control this quality margin by grading loans according to quality (unpublished study by T. O. Yntema).

These other margins of cost may be exceedingly important for certain types of policies since the rational basis for any business decision may be viewed as a comparison of the anticipated margins. Marginal cost should be balanced against the marginal revenue attributable to a given order or a given course of action. The marginal cost of one method is also to be compared with the incremental cost of performing the service in a different way, and much business thinking goes on at this level without reference to the cost-revenue marginal equality. In either case, estimates of the differential costs of alternatives are needed, and these alternatives frequently involve the computation of margins of cost for other operating conditions in addition to output.¹⁷

Frequently, several of these margins must be considered in combination for a single decision. If the decision in question involves an increase of both output and the number of new styles, for instance, then the increment in total cost due to new styles must be added to the increment caused by additional output. To put it more generally, since cost is a function of several variables:

$$X_T = f(X_2 X_3 X_4 \dots X_n),$$

the addition to cost which is pertinent to a decision involving changes in several of these factors is the sum of the increments attributable to these various factors, i.e.,

$$dX_T = \frac{\partial f}{\partial X_2} dX_2 + \frac{\partial f}{\partial X_3} dX_3 + \frac{\partial f}{\partial X_4} dX_4 \dots \frac{\partial f}{\partial X_n} dX_n.$$

Price theory usually ignores all these cost increments except that for quantity of output.¹⁸ This simplification is correct when other margins are unimportant or when decisions can be formed in such a way as to involve changes only in one operating condition at a time.¹⁹ For many

¹⁷ It should be pointed out that these increments need not be computed with great precision for, as a rule, the estimates with which they are compared are subject to such a wide margin of error that meticulous refinement of these cost margins is not worth while.

¹⁸ Statistical exploration of this type is useful in calling attention to the limitations of the simplified assumptions of the neo-classical price theory.

¹⁹ The level at which the other factors are held constant may also have some effect upon the magnitude of the increment for the varied factor. Such information is not provided by this study since the independent variables were, in effect, held constant at their means.

practical pricing decisions, however, this approach is too simple since simultaneous change in several operating conditions is involved and summation of their respective cost increments is therefore required. For example, the particular firm studied has important orders from large mail-order houses. The acceptance of one of these orders often involves, in addition to an increase in the quantity of output produced, the design and production of new styles, increases in the average size of production order, sharp changes in the level of output from that of the previous period, and some change in the rate of labor turnover. Changes in each of these important conditions of operation involves some increment (or decrement) in cost. Consequently, to ignore all except the output increment might involve serious error.²⁰

COMPUTATION OF OTHER MARGINS OF COMBINED COST

Several conditions must be satisfied before it is possible to compute the marginal cost of varying other operating factors. First, a numerical measure of the factor must be available; otherwise it cannot be treated mathematically. Second, the factor in question must be an independent variable; that is, the proportion in which this cost factor is combined with other operating conditions must be in some degree variable. Without this flexibility it is not possible to measure the partial derivative, and, even if this derivative were measured, it would not be possible for management to profit by this knowledge.

Rough, provisional estimates were made of margins of combined cost for four operating conditions: size of production order (X_3), new styles (X_4), changes in rate of output (X_5), and rate of labor turnover (X_7). These computations were made more to indicate the nature of the problem than to provide a final solution for it.

The procedure followed is similar to that used for calculating the output margin. The average slope of the net regression curve for size of production order and cost per unit of output (Fig. 2) was obtained graphically by fitting a straight line freehand to the net residuals of the multiple correlation analysis. This slope ($-.004$) was taken as

²⁰ Even allowance for these other margins of cost is occasionally insufficient since a particular order may require special additional expenses (such as retooling) over and above those included in the typical increments of cost.

the approximate partial derivative of the average cost function.²¹ It was transformed into the partial derivative of the total cost function by using the following relationship:

$$\begin{aligned} X_A &= f(X_3) && \text{(net regression line for average cost)} \\ X_T &= X_2 \cdot f(X_3) && \text{(net regression line for total cost)} \\ \frac{\partial X_T}{\partial X_3} &= \partial \frac{[X_2 \cdot f(X_3)]}{dX_3} \end{aligned}$$

Assuming that the derivative of output with respect to the size of production order is zero, then

$$\begin{aligned} \frac{\partial X_T}{\partial X_3} &= X_2 \cdot \frac{df(X_3)}{dX_3} \\ &= (\text{Rate of output}) \times (\text{Slope of net regression for} \\ &\quad \text{production order}) . \end{aligned}$$

By similar procedures the following estimates of other increments of cost were obtained:

Size of production order (X_3):²²

$$\frac{\partial X_T}{\partial X_3} = X_2 \cdot (-\$0.004) \quad \begin{array}{l} \text{(for an increase of one case in average size} \\ \text{of production order)} \end{array}$$

Number of new styles (X_4):

$$\frac{\partial X_T}{\partial X_4} = X_2 \cdot (+\$0.0055) \quad \text{(for an increase of one new style per period)}$$

Change in rate of output (X_5):

$$\frac{\partial X_T}{\partial X_5} = X_2 \cdot (-\$0.0044) \quad \begin{array}{l} \text{(for an increase in output above the pre-} \\ \text{vious period of \$1,000 O.W.V.)} \end{array}$$

²¹ Since the distribution of residuals was approximately linear and since insufficient a priori grounds could be found for a curvilinear regression, a straight line was thought to be sufficiently accurate for the purpose.

²² This margin should be negative since it represents the cost increment of making the same output in larger rather than in smaller lots. The proportionately variable costs would have no marginal cost since they would be constant, whereas the fixed costs of the production order would decline, since there would be fewer setups.

Labor turnover (X_7):

$$\frac{\partial X_T}{\partial X_7} = X_2 \cdot (+\$0.11) \quad (\text{for an increase of 1 per cent in labor turnover for accounting period})$$

Estimates of curvilinear marginal cost were also made for two independent variables: size of production order (X_3) and number of new styles (X_4). These functions are shown in chapter viii of the author's dissertation previously cited.

The following example is given of methods of combining the various constant margins of cost into a single estimate of the addition to cost consequent upon acceptance of an order which changes three conditions of operation— X_2 , X_3 , and X_4 .

Since

$$X_T = f(X_2 X_3 X_4),$$

then

$$dX_T = \frac{\partial f}{\partial X_2} dX_2 + \frac{\partial f}{\partial X_3} dX_3 + \frac{\partial f}{\partial X_4} dX_4.$$

If the partial derivatives are constant at the values given below, if the contemplated increment of each variable is small, if other factors than $X_2 X_3$ and X_4 are negligible, and if

$$dX_2 = 1 \text{ (dollar old warehouse value)}$$

$$dX_3 = 1 \text{ (case production order)}$$

$$dX_4 = 1 \text{ (new style per accounting period)}$$

and

$$\frac{\partial f}{\partial X_2} = \$1.116$$

$$\frac{\partial f}{\partial X_3} = -.004$$

$$\frac{\partial f}{\partial X_4} = +.0055$$

then

$$\begin{aligned} dX_T &= (\$1.116) \cdot (1) + (-\$0.004) \cdot (1) + (\$0.0055) \cdot (1) = \$1.1775 \\ &= \text{additional cost for this decision.} \end{aligned}$$

If, on the other hand, the incremental functions are curves, rather than constants, the problem is not greatly altered since it is then simply necessary to evaluate the incremental functions for the appropriate value of each independent variable. If the total cost is correlated jointly with the various independent variables, the determination of the incremental costs is, of course, much more complex.

CHAPTER VIII

PRACTICAL APPLICATIONS OF FINDINGS

The purpose of this chapter is to indicate the practical uses to which the findings of this study can be put and to show how to convert the average and the marginal cost estimates discussed in chapters vi and vii and in Appendixes A and B into a form which will serve these practical ends. The discussion is divided into two parts, the first of which deals with average cost findings and the second with marginal cost findings.

AVERAGE COST FINDINGS

PRACTICAL USES FOR AVERAGE COST FINDINGS

The most important practical use to which the average cost findings of this study can be put is in the determination of flexible standards for cost control. Control of expenses is often inadequate when the customary fixed standards are relied upon because these expense standards make no allowance for the effect upon cost of changes in operating conditions. Flexible cost standards which are free from these defects can be developed from the net regression curves of the multiple correlation analyses of combined cost found in chapter vi and in Appendix A.¹ By reading from these curves standard costs can be determined which will reflect the actual operating conditions of the period (e.g., rate of output, size of production order, number of new styles, etc.). Cost criteria thus developed are superior as control devices because a departure from the flexible standard is more strictly due to variation in efficiency than to any other factor.²

This type of control device deserves more general use than it now enjoys. Cost is commonly a function of other less uncontrollable factors as well as of managerial efficiency. Hence these other relation-

¹ See also chap. viii and Appendix C of the author's dissertation, "A Statistical Examination of the Behavior of Average and Marginal Costs," for a more detailed discussion of the regression curves of the constituents of combined cost.

² Livingston advocates the use of the gross regression line of a linear correlation analysis as a standard of expense (*Control of Operating Expenses*, p. 716).

ships must be recognized if expense standards which will deal justly with executives are to be set up.

In addition to this important application of average cost findings there are certain minor purposes for which these cost estimates may be used. One of these is to evaluate proposals for reducing cost. Another is to aid in forecasting future expenditures and profits under anticipated operating conditions.³

The average cost findings of this study have not been presented in a form which will best serve the practical ends discussed above. To achieve these purposes more effectively certain adjustments must be made in the regression curve magnitudes. The cost estimates must be placed on a parity with current (or future) cost with respect to (1) the level of input prices and (2) the lag and lead correction.

The adjustment of these estimates to correspond to the current level of prices is necessary because the cost data from which these curves were derived were reduced to the price level of January, 1932. Price adjustments of this type can be readily made, either upon the basis of a composite weighted index number of the prices of the input factors, or upon the basis of individual price indexes applied separately to each of the components of combined cost. The former method is simpler but may be less accurate, partly because the weighted average of the composite index becomes less representative as the lapse of time between the corrected period and the base period increases. Here separate correction of the components will probably be more satisfactory since estimates of component elements must be corrected individually if they are to be used for purposes of control or forecast. Individual correction will not be costly since indexes for the components have already been worked out and need only be kept up to date.

In addition to adjusting the cost estimates so as to make them correspond with the current level of prices these estimates must also be made comparable with current costs with respect to lag and lead correction. Cost data used in developing the estimates were adjusted so as to correspond to the output to which the actual cost contributed. Since, on the other hand, most cost entries anticipate output, the recorded cost of a current month has not been thus adjusted and hence

³ These minor uses have been discussed in chap. vi.

does not correspond exactly to the recorded output of that period. Therefore, either the curve estimate or the current figure must be adjusted if the two are to be strictly comparable. If the cost estimate is to be used to predict future profits, the lag correction should not be removed, for the cost estimate, in this instance, should remain associated with the actual output to which it contributed. If, on the other hand, the objective is to compare present recorded cost with a flexible standard developed from the regression curves, then the recorded cost with which the estimates are being contrasted should be subjected to the same lag correction as the estimated standard.

In making the adjustments for output lag two methods are available. (1) Combined cost for the current period can be corrected as a whole by a weighted average of the time lag adjustments employed for its separate component elements. (2) Each constituent of current cost can be adjusted separately by the lag correction which was developed for rectifying the original data. The former procedure appears to be simpler, but the latter will be preferred if the cost components are to be used separately for purposes of control or prediction.

DEVICES WHICH FACILITATE THE USE OF AVERAGE COST FINDINGS

To maximize the practical usefulness of the average cost findings of such a study as this not only is it desirable to convert the cost estimates into comparable form (with respect to price level and output lag), but it is also important to call attention to devices for recording and manipulating these cost estimates—devices which will facilitate the use of these findings and make it convenient for management to apply them. These devices may be grouped as follows:

1. Net regression curves
2. Tables of estimated cost under various operating conditions
3. Charts relating costs to income at various levels of output
4. Flexible budgets

The net regression curves of the multiple correlation analyses are themselves a convenient and flexible record of results. Algebraic summation of readings from a set of these curves for any combination of current or anticipated operating conditions gives a rough estimate of average cost. These regression curves can also be converted into a second device for presenting results, namely, classified tables which

show estimated cost under various operating conditions. Such a table was constructed for combined cost as a convenience to the management of the particular furniture factory studied (Table I).

A third type of device is the break-even chart which compares cost and income at various levels of output. To construct such a chart the expected total magnitude of combined cost and the expected total net sales revenue are plotted as the ordinate, and the rate of output as the abscissa. The distance between the total cost line and the income line represents profits or losses.⁴

Visual administration of flexible cost standards by calling attention to departures of current cost from the net or gross regression curve which is used as the performance standard is a promising development. Two-dimensional charting, however, makes budgets flexible with respect to only one operating condition at a time. Such simplification is warranted only when this one variable is of dominant importance. Budgets which are flexible in a fuller sense of the term must be adjusted simultaneously for the effect of several operating conditions and must be revamped as the expectations with regard to these conditions change. The labor of constructing and revising such a budget can be reduced by tabulating on permanent data sheets the cost adjustments for a series of values of each operating condition read from the net regression curves.

MARGINAL COST FINDINGS

The practical application of marginal cost findings plays a subordinate rôle in a study whose principal concern is with methods of determining marginal cost rather than with ways of using them. The discussion which follows is limited, therefore, to the three types of considerations which affect the practical usefulness of marginal cost estimates in an individual firm.

1. Uses to which marginal cost findings can be put
2. Problems of transforming the marginal cost estimates of this study into a form which is readily usable for these purposes
3. Limitations upon the utilization of these transformed marginal cost estimates for practical business decisions

⁴ For a brief description of the break-even chart see Rautenstrauch, *The Economic Characteristics of Manufacturing Industries*, p. 223. Note that allowance is made for only one operating condition, namely, output.

PRACTICAL USES FOR MARGINAL COST ESTIMATES

The principal use to which marginal cost estimates can be put is to aid in determining the most profitable price and output policy. The selling price which yields the highest profit can be determined, within limits to be discussed later, from the estimated future marginal cost curve when this curve is considered in conjunction with the future marginal revenue curve. This maximum profit price is the price which corresponds to the output at which marginal cost and marginal revenue are equated.⁵

Even when the demand schedule and marginal revenue curve of the firm in question are not known precisely, marginal cost may be of some service in determining a rational price policy. If the market is sectionalized and a series of class prices prevails, then the marginal cost sets a limit below which the firm cannot, except under unusual circumstances, afford to allow its lowest class price to drop. For example, if chain stores represent the minimum price sector of the market, then the lowest price at which it will be profitable to sell to that sector will be the incremental cost of these sales (sales at higher prices, to other sectors of the market, are assumed to be unaffected).⁶ Since such buyers often bid on the basis of assumed knowledge of the differential cost of manufacture, it is well for the producer to know what his incremental cost actually is.

Another closely associated use for marginal cost estimates is to help determine the wisdom of introducing a new product or of varying the proportion of the divers articles manufactured. When the marginal costs of various ways of modifying a product are known, and when the probable increment in revenue resulting therefrom can be estimated with any degree of accuracy, then the equating of these two estimates provides a rational guide to the most profitable type of alteration of the product. In like manner marginal cost is the rational criterion for deciding upon the proportion of the various articles which should be manufactured. This guide should, however, be supplemented by other

⁵ See Jacob Viner, "Cost," *Encyclopedia of Social Sciences*, p. 467. Frequently considerations other than maximization of immediate profit influence price policy.

⁶ If the market is not perfectly sectionalized, i.e., if sales to chain stores involve loss of sales through other channels, then the lowest acceptable price will be higher than marginal cost since the marginal revenue, which must not be lower than marginal cost, will then be less than the price.

information since estimates of marginal cost are not allocated to particular products with great accuracy, and since increments in revenue attributable to the various articles and combinations of articles can be estimated only within wide margin of error. Nevertheless, both these margins can be estimated with sufficient accuracy to make such studies highly useful for this important purpose.

The marginal cost findings of this study are not usable as a guide to the proper proportioning of the factors of production, for the components of marginal combined cost do not represent partial derivatives of the total cost function and, therefore, are not measurements of the marginal productivity of the factors.

ADJUSTING MARGINAL COST ESTIMATES

To make the marginal cost estimates of this study usable for the objectives discussed above they must be adjusted in several ways. First, they must be corrected for those changes in the prices of input factors which have occurred between the base period of the analysis (January, 1932) and the time at which these estimates are being applied. This adjustment can be made by methods analogous to those suggested for correcting estimates of average cost.

A second type of adjustment consists of recognizing the effect upon marginal cost of operating conditions other than the rate of output.⁷ Since total cost is a function of many variables it has many margins—one for each factor with which it varies. Frequently several of these margins, in addition to the commonly recognized output margin, are of quantitative significance. In the particular firm studied, as has been noted in chapter vii, the cost increments associated with increasing the size of lot, the number of new styles, and the rate of labor turnover have been roughly measured. When acceptance of a given order involves change in these other operating conditions, then their respective increments of cost must be added to the marginal cost of increasing the output in order to obtain the actual increment in cost caused by the order in question.

The allocation of marginal cost to particular products is a third type of adjustment. This disposition is especially important in getting the

⁷ In computing marginal cost the other operating conditions were, in effect, held constant at their respective means. The level at which they are held constant may have some effect upon the marginal cost function. This effect was not investigated in this study.

cost estimates into usable form because price and output decisions are commonly concerned with particular types and articles of product, rather than with output in general.

Typical practical questions which arise are: whether or not to accept an order for a given style at a specific price; whether or not to alter an article in a particular fashion in order to make it more salable; whether or not to place more stress on a particular line of products in the coming sales campaign; and the like. In dealing with these problems the incremental costs of particular articles of product are of dominant interest to the management. Some method, therefore, must be devised for attaching the marginal cost of general output to specific articles of product. At first sight the task of attaching the marginal cost of general output to specific articles and lots of produce appears to be hopeless. Theoretically, the only accurate solution for the problem is to vary the proportions of the several products freely and extensively, and then study the effects upon the total cost resulting from increasing the amount of product A while holding constant, in various proportions, products B, C, and D. Such a procedure is, of course, quite out of the question when there are, as in the present case, hundreds of products. Therefore, some rough, practical approximation to this theoretically correct solution must be devised.

The simplest approximation is to allocate marginal combined cost to the article in question on the basis of its old warehouse value. Suppose that it is necessary to determine the minimum price at which a particular order for one hundred units of a specific model of furniture can be accepted. First the total old warehouse value (deflated standard cost) of this order will be computed, say at \$1,000. Second, this value will be multiplied by the estimated marginal cost per dollar of old warehouse value at the expected level of output, perhaps \$1.12, giving a marginal cost of \$1,120 for the order. This estimate, in turn, can then be adjusted if necessary for the forecast price level and for the effect of other cost influences. Although it requires some rather questionable assumptions,⁸ this method's simplicity and economy recommend it highly when dealing with marginal cost estimates of limited accuracy.

⁸ This average method assumes, first, that the old warehouse value of the article under discussion is a correct measure of output and, second, that this particular article corresponds to the average of the products manufactured with respect to the

Another method of estimating the marginal cost of a particular article is to build up the combined marginal cost for this item from its various components by combining the cost constituents in the proportion peculiar to this particular article of product. This method is described in considerable detail in Appendix B. There is some question, however, whether this additive method will give more accurate marginal cost estimates than will the simpler procedure of allocating combined marginal cost according to the old warehouse value of the item.⁹

Even when marginal cost estimates have been correctly converted into usable form, their value is further restricted by other limitations. First, the expense of securing information of this type may put it beyond the reach of most firms. Second, shifts in cost relationships caused by technological progress, reorganization of production routines, etc., may quickly render expensively computed marginal cost estimates obsolete. Third, lack of accurate estimates of marginal revenue may seriously limit the usefulness of estimates of marginal cost.

Finally, there are certain imponderables such as the effect upon trade connections, attitude of customers, morale of the working force, etc., which, although they cannot be measured, nevertheless do influence rational price policy decisions. Hence, even with accurate information regarding incremental revenue and incremental cost, there remain wide margins of uncertainty within which habit, business philosophy, and social pressure will sway decisions.

behavior and the relative importance of its cost constituents. Only under such conditions could the computed average ratio of marginal cost to old warehouse value be validly applied to any particular article of furniture. See Appendix B for discussion of these assumptions.

⁹ In the first place, it should be remembered that these components of marginal cost are not partial derivatives of the total cost function; they represent, instead, the increment of cost of the particular component which is associated with a unit increase in output with other variable cost items increasing as they typically do. In the second place, it is assumed that old warehouse value measures output accurately and that standard average cost and its components are satisfactory indicators of the proportions of the cost constituents. Third, we assume that the ratio of marginal cost under any particular set of operating conditions to average cost under standard or normal conditions is the same for all articles manufactured. Fourth, it is supposed that for any given size and combination of output the proportions of cost constituents are fixed within rather narrow limits. These assumptions are discussed more fully in Appendix B.

In conclusion, it should be pointed out that in the practical use of marginal cost estimates great care should be given to the determination of the time period which is really appropriate for a given decision. Many policies which properly involve long-period commitments and ramifications are incorrectly formulated in the light of short-period considerations (and vice versa). For numerous types of decisions an intermediate-period rather than a short-period or a long-period marginal cost is required, since the psychological, social, and economic repercussions of the decision stretch forward some distance into the future. Rough estimates of the required intermediate-period marginal cost can be made on the basis of the findings of this study by taking a figure somewhere between the future short-period marginal cost¹⁰ and the extrapolated average cost (which often constitutes a moderately good approximation to long-period marginal cost).

SUMMARY

In order to facilitate the practical application of these average cost findings certain commercial uses have been pointed out. The most important managerial benefits are: (1) the determination of flexible cost standards which will make expense control more effective; (2) a saner evaluation of proposals for reducing cost—the result of more complete knowledge of the forces which affect cost behavior; and (3) a more accurate forecasting of expenditures and profits.

To put the cost findings into a form which will serve these managerial ends these estimates must be made comparable with current (or future) costs with respect both to the level of input prices and to the correction for lag and lead. Both of these adjustments can be made either by means of a composite correction index (for price level and for output lag) or by means of a separate adjustment of the individual components of combined cost. The latter method seems to be preferable for both types of cost adjustments since the components are to be used separately for control and for prediction purposes.

To facilitate the use of these adjusted cost estimates attention has been called to certain devices which make the findings available in

¹⁰ The need for extrapolating any cost estimate used in making policy decisions should be obvious. Estimates of past marginal cost are of managerial interest solely as a basis for predicting future marginal cost, for it is this extrapolated cost which is balanced against expected marginal revenue.

convenient form. The most useful of these are the net regression curves of the correlation analysis, tabular summaries of these curves, break-even charts compiled from adjusted estimates, and permanent data sheets for flexible budgets.

The major practical use for marginal cost estimates is to determine the most profitable price and output policy. But this use has wide ramifications since a large proportion of business decisions can and should be put in terms of balancing the added cost of a certain policy against the added revenue resulting from this step. The findings of this study are not sufficiently refined, however, for use in many of these types of decisions.

Three types of adjustments must be made in the marginal cost estimates before they are usable for the practical purposes just mentioned. The adjustments are:

1. Marginal cost estimates should be adjusted to correspond to the price level of the period in which the estimates are to be used.
2. These corrected estimates should take into consideration the cost margins of other factors, such as size of production order, number of new styles, etc., as well as the marginal cost of increasing output.
3. The adjusted marginal cost estimates must be allocated to particular articles of product. This allocation can be accomplished in a rough way by assigning marginal cost to a particular article in proportion to that item's old warehouse value. A method which is more laborious, but may not be more accurate, is to build up the combined incremental cost of an article from the components of combined marginal cost.

Even if these adjustments are properly made, the usefulness of these marginal cost estimates is restricted by certain additional limitations: the expense of securing incremental cost figures and the limited life of these estimates owing to changes in fundamental cost conditions, the lack of information concerning marginal revenue, and the presence of certain imponderable considerations which must also enter into rational price decisions.

But these qualifications must not obscure the fundamental need of business men for better estimates of marginal costs and the importance of improving methods for making such estimates.

APPENDIX A

BEHAVIOR OF COMPONENTS OF AVERAGE COMBINED COST

The behavior of the average and total cost curves of constituent elements of combined cost is, for several reasons, of considerable interest to management. The most important service rendered by such knowledge is that it aids in the effective control of cost by providing the basis for setting up flexible cost standards in sufficient detail to be managerially usable. A second benefit derived from this detailed knowledge of the behavior patterns of cost elements is that such information makes possible more specific and exact allowances for dynamic changes such as shifts in particular prices or wage rates and minor alterations of production technology. Such allowances tend to increase the accuracy of predictions of combined cost. A third reason for managerial interest in the findings of this chapter is the light which they throw upon proposals for cost reduction and for alteration of production routines. A more exact appraisal of the effect of such policies can be made if the knowledge of combined cost is supplemented by an analysis of the behavior of its component elements. Finally, the analysis of the **average** or total cost behavior of these constituents by means of multiple correlation is a highly desirable preliminary step in the reliable estimation of the components of marginal combined cost--a type of information which enhances the flexibility and accuracy of marginal cost estimates.

In using these multiple correlation results for the foregoing purposes the method and the underlying assumption of the separate analysis of these components must be kept in mind. The method followed was to take the actual recorded expenditure for each component without attempting to hold constant the expenditures for other components. All components were allowed to vary together in the manner which the management had regarded as the most advisable combination in the light of the operating conditions of the period. The recorded expenditure for each component was separately corrected for price changes and for output lag and

was then subjected to a graphic multiple correlation analysis. The basic assumption which underlay this procedure was that negligible substitution between cost components took place as a result of changes in their relative prices (pronounced though these price changes were during this period). However, substitution as a result of changes in the volume of output and in other operating conditions was fully reflected and, consequently, must be regarded as the major cause for the observed cost behavior. Thus, we have a dynamic record of the proportion in which management saw fit to combine resources in response to short-period changes in output, size of production order, number of new styles, and the like.¹

Direct Labor Cost

The net regression curve for output (Fig. 9) indicates by its continuous decline that direct labor cost per unit diminishes as the rate of production activity increases. Such cost behavior appears to be in accord with expectations.² The second regression of Figure 9 indicates that direct labor cost per unit also diminishes when the product is manufactured in larger lots. Such behavior is to be expected in view of the high ratio of setup time to running time in the machine operations of this factory, for the spreading of a fixed setup cost over a large number of pieces results in a lowering of the cost per piece. In accord with reasonable expectations the middle curve of Figure 9 shows that average labor cost rises at a uniform rate as the number of new styles is increased; new models usually involve special instruction of workers and are likely to cause confusion in production

¹It should be pointed out that the average cost curves of components therefore do not precisely represent what the economist calls the "technical coefficients of production," for these curves do not show the cost of a particular input factor per unit of output when all other input components have been held constant. These curves may, nevertheless, give some rough indication of the behavior of these technical coefficients.

²Direct labor cost, though labeled "direct," is not a purely variable cost but contains elements of fixed cost. Moreover, labor tends to be more highly specialized and better organized as the plant approaches the output levels for which it was designed. Output had apparently not reached (in the range of observations included) the point at which the operation of the law of diminishing returns begins to cause cost increases. Psychological and organizational factors may more than offset this latter tendency over the practical range of output.

PARTIAL REGRESSION CURVES OF AVERAGE DIRECT LABOR COST PER UNIT WITH FIVE INDEPENDENT FACTORS: OUTPUT (X_2), SIZE OF PRODUCTION ORDER (X_3), NEW STYLES (X_4), PRODUCTION INCREASES (X_5), AND LABOR TURNOVER (X_7),

X_A AVERAGE DIRECT LABOR COST PER UNIT

ADJUSTMENTS IN AVERAGE DIRECT LABOR COST PER UNIT

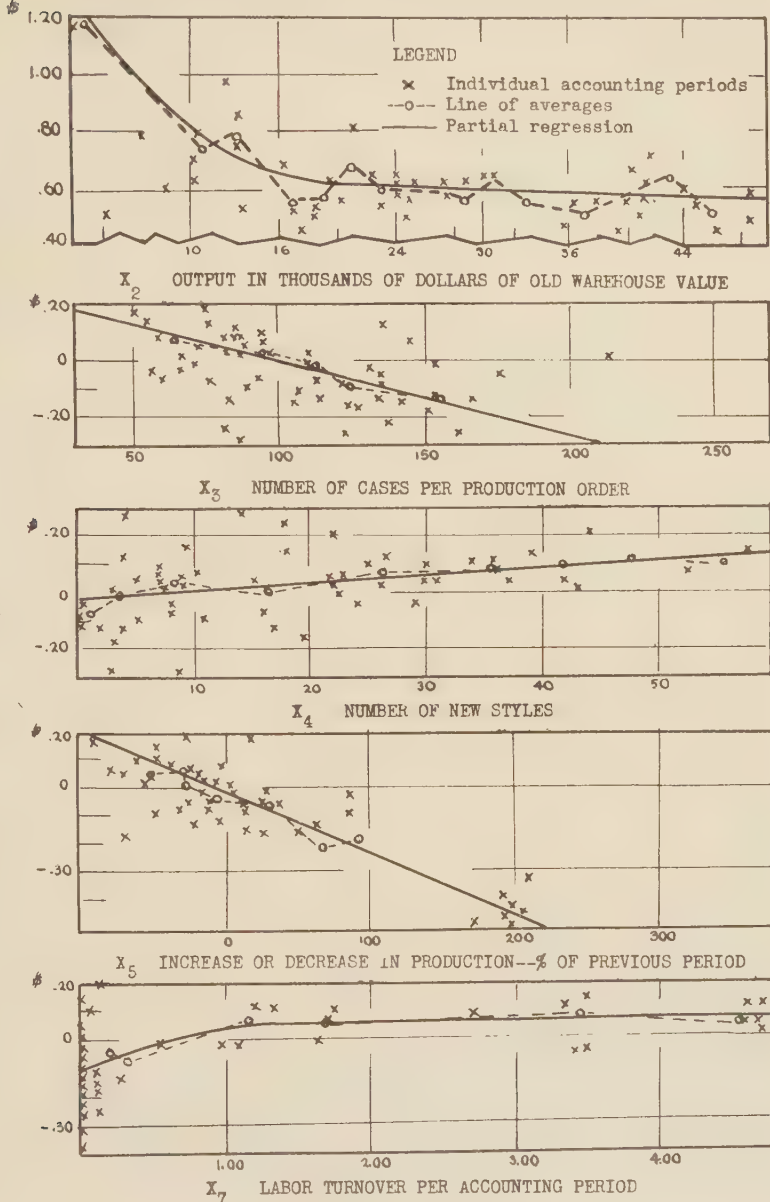


FIG. 9

routines. The fourth curve shows that a striking increase in labor cost per unit accompanies a reduction in the level of output from that of the previous accounting period. This behavior may be partly caused by failure to reduce and reorganize the working force for the new low level of production. The bottom curve of Figure 9 shows, as would be expected, that higher labor turnover is associated with high labor cost.

TABLE VI

MEASURE OF THE UNRELIABILITY OF AVERAGE
DIRECT LABOR COST FUNCTION*

(Standard error of difference between partial
regression curve at specified values of
output and partial regression curve
at average value of output)

Output (Unit \$1,000 in O.W.V.)	Estimated Cost		Standard Error of Deviations in Col. 3†
	Reading from Curve	Deviation from Reading at Mean Output†	
(1)	(2)	(3)	(4)
5.....	\$1.070	\$.483	\$.077
10.....	0.814	.227	.056
15.....	0.672	.085	.039
20.....	0.613	.026	.023
25.....	0.597	.010	.020
30.....	0.588	.001	.005
35.....	0.580	- .007	.025
40.....	0.572	- .015	.031
45.....	0.563	- .024	.060
50.....	0.550	- .037	.098

* The method of computing this standard error is described in Ezekiel, *Methods of Correlation Analysis*, p. 384. The formulas are provisional and give only a rough approximation.

† Departures of curve readings from the curve value \$0.587, which corresponds to the mean of output (\$29,720).

‡ Standard error refers to departures described in the preceding footnote.

On the basis of the standard error of departures of the output regression curve from its value at the mean level of output (shown in Table VI) the following conclusions were reached:

(1) For the outputs greater than \$25,000, O.W.V., little statistical significance can be attached to departures of this curve from its reading at the mean of output. (2) Only a moderate degree of confidence can be placed in the curve departures in the vicinity of \$20,000 output. (3) Considerable significance can be attached to the curve below \$20,000 output. It must be remembered, nevertheless, that the reliability of this curve as a basis for estimating cost behavior is not measured by these standard errors.

TABLE VII

RELATIVE IMPORTANCE OF THE SEVERAL INDEPENDENT
FACTORS INFLUENCING LABOR COST AS SHOWN BY
THEIR COEFFICIENTS OF NET REGRESSION,
COEFFICIENTS OF PART CORRELATION,
AND BETA COEFFICIENTS

Independent Factor	Coefficient of Net Regression ($b_{12.3457}$, etc.)	Coefficients of Part Corre- lation (12^r_{3457} , etc.)	Beta Coefficients ($B_{12.3457}$, etc.)
X_2 (output).....	Curve	.905	Curve
X_3 (order size).	.00266	.54	.317
X_4 (new styles).	.002408	.31	.163
X_5 (change in output)....	.00226	.67	.445
X_7 (labor turn- over).....	Curve	.18	Curve

Other conditions revealed by the correlation analysis indicate that the net regression curves may be used with considerable confidence in setting standards and in forecasting results, provided, of course, that the underlying conditions remain the same as during the period of study. Each of the net regressions indicated cost behavior which was sensible. Used together, these curves accounted for a very large proportion of cost variance (75.2 per cent). Moreover, cost estimates made from them had a narrow band of standard error (\$.108) and were highly correlated with actual labor cost (multiple correlation index of .867); see Table II.

Rough measurements were also made of the relative importance of the influences of the several independent factors upon average direct labor cost. These indexes are shown in Table VII. As indicated by the somewhat unreliable coefficients of part correlation, output ($\bar{X}_2$) was the most important factor (.905); change in production ($\bar{X}_5$) was second (.67); size of lot ($\bar{X}_3$) third (.54); new styles ($\bar{X}_4$) fourth (.31); and labor turnover ($\bar{X}_7$) had least importance. This hierarchy of relative importance is substantiated by the Beta coefficients and is, moreover, in general accord with reasonable expectations. The importance of changes in output ($\bar{X}_5$) is somewhat surprising, and the relatively small effect of the rate of labor turnover ($\bar{X}_7$) upon direct labor cost will be of interest to personnel managers. It is dangerous, however, to generalize from the observed cost behavior of one firm.

Indirect Labor Cost

The behavior of average indirect labor cost which is indicated by the net regression curves of Figure 10 is, on the whole, in accord with a priori expectations. This partially fixed cost falls continuously, at a diminishing rate, as output increases. The number of new styles appeared to have no clear influence upon indirect labor cost. However, changes in output from that of the previous period had the expected effect: Decreases in output caused larger per-unit magnitudes of this relatively inflexible cost whereas small increases were associated with smaller cost since this inflexibility then worked to the firm's advantage.

These curvilinear paths of cost behavior accounted for about 90 per cent of the total variance of this cost item, according to the index of multiple determination. Estimates of cost made on the basis of these curves were highly correlated with the actual cost values as indicated by the coefficient of multiple correlation, .949 (see Table II). The standard error of estimate for indirect labor cost was \$0.022, which compares favorably with the standard deviation, \$0.069. From these several measures it appears that a good deal of confidence can be placed in these regression results in accounting for the behavior of average indirect labor cost.

The standard error of departures of the output regression curve from the curve value at the output mean are shown in Table VIII.

PARTIAL REGRESSION CURVES OF AVERAGE INDIRECT LABOR COST PER UNIT
FOR THREE INDEPENDENT FACTORS: OUTPUT (X_2), NUMBER OF NEW
STYLES (X_4), AND INCREASES IN PRODUCTION (X_5)

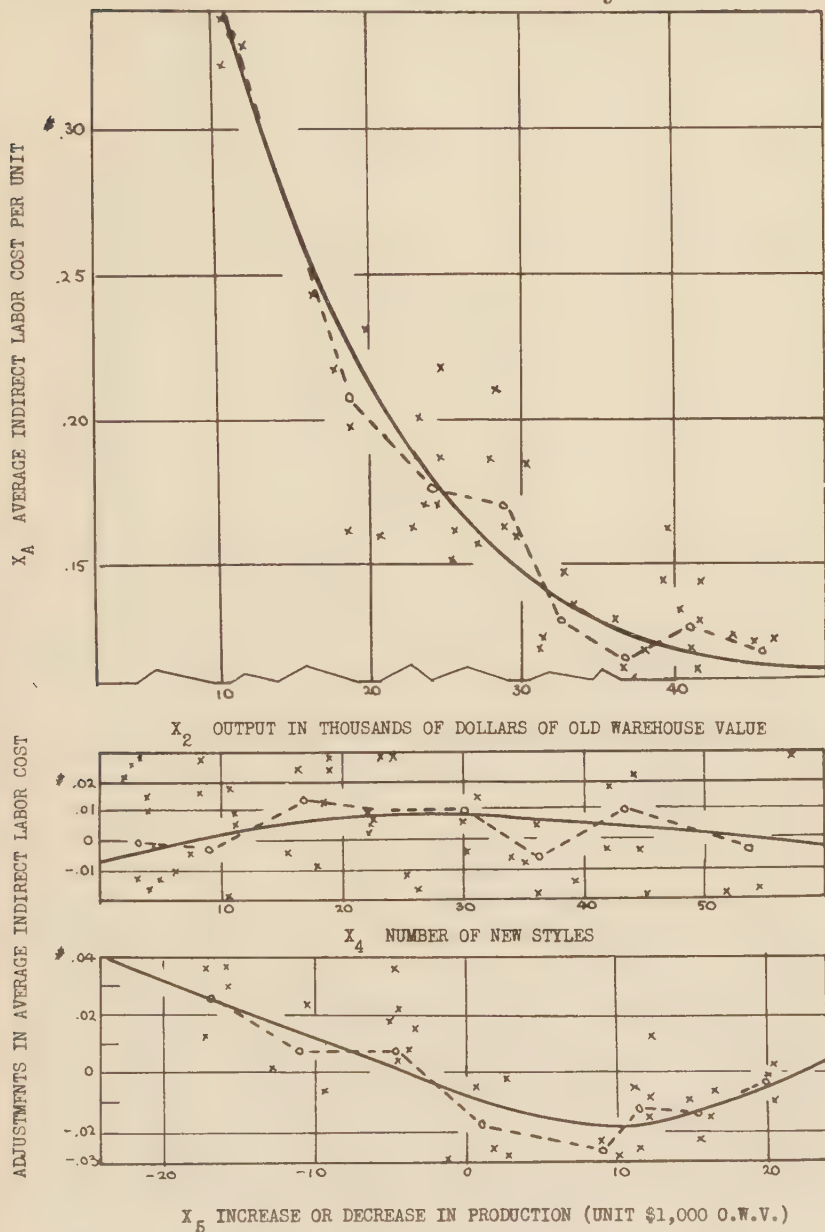


FIG. 10

TABLE VIII

MEASURE OF THE UNRELIABILITY OF THE AVERAGE
INDIRECT LABOR COST FUNCTION*

(Standard error of difference between partial
regression curve at specified values of
output and partial regression curve
at average value of output)

Output (Unit \$1,000 in O.W.V.)	Estimated Cost		Standard Error of Deviations in Col. 3†
	Reading from Curve	Deviation from Reading at Mean Output‡	
(1)	(2)	(3)	(4)
10.....	\$.3590	\$.2113	\$.0362
15.....	.2665	.1188	.0442
20.....	.2104	.0627	.0207
25.....	.1735	.0258	.0107
30.....	.1466	- .0011	.0029
35.....	.1288	- .0189	.0142
40.....	.1198	- .0279	.0174
45.....	.1151	- .0326	.0285
50.....	.1140	- .0337	.0734

*The method of computing this standard error is described in Ezekiel, *Methods of Correlation Analysis*, p. 384. The formulas are provisional and give only a rough approximation.

‡Departures of curve readings from the curve value \$0.1477, which corresponds to the mean of output (\$29,720).

†Standard error refers to departures described in the preceding footnote.

Inspection Labor Cost

The surprising characteristic of total inspection labor cost which is brought out by its output regression curve of Figure 11 is its flexibility. This flexibility was apparently brought about by transfer or partial layoff of inspectors. In the second chart of this figure it is indicated that large manufacturing lots, reasonably enough, bring a reduction in the total inspection cost. But increases in the number of new styles are not associated with the expected increase in this cost but, instead, with a strange decrease. Perhaps the inspection task for newly introduced styles is really performed by foremen, engineers, and workmen, thus leaving less rather than more for formal

PARTIAL REGRESSION CURVES OF INSPECTION LABOR TOTAL COST FOR
FOUR INDEPENDENT FACTORS: OUTPUT (X_2), SIZE OF PRODUCTION
ORDER (X_3), NUMBER OF NEW STYLES (X_4), AND QUALITY
OF MATERIAL (X_8)

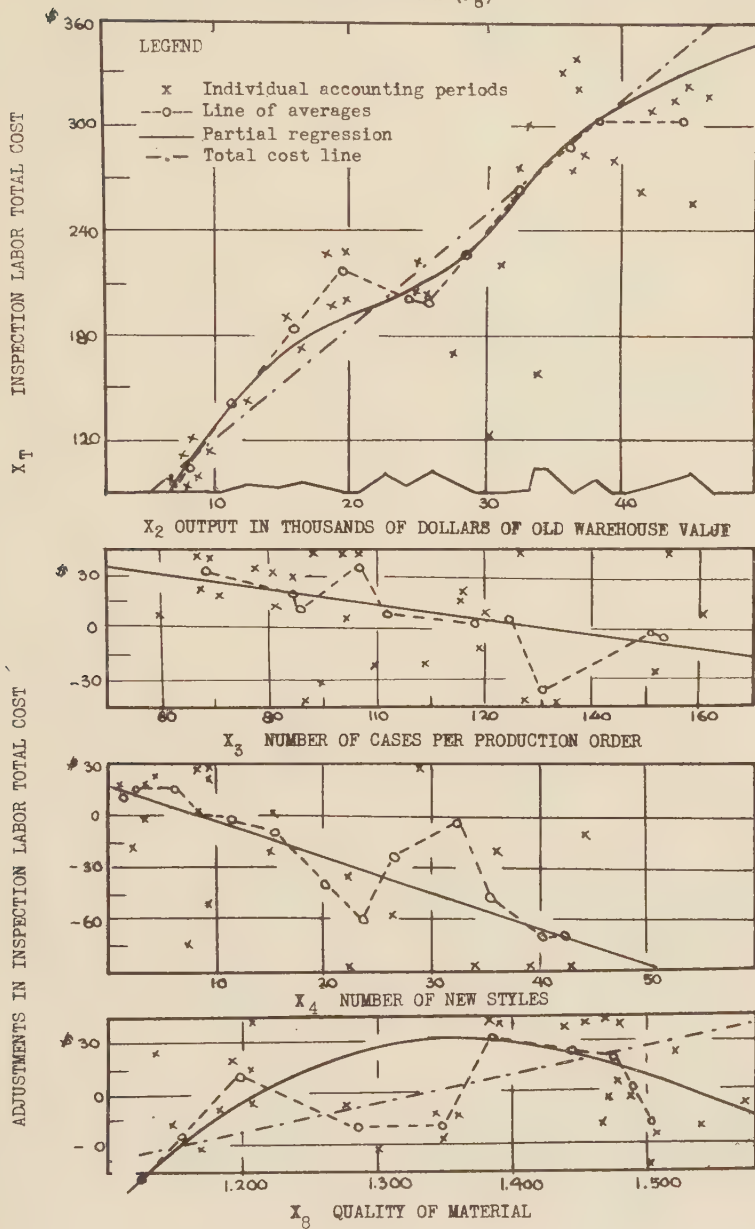


FIG. 11

inspection. The curve for quality of materials also shows a somewhat illogical cost behavior. Cost should rise continuously, instead of first rising and then falling off for high qualities. Perhaps the illogical shape of this regression is explained by the poor curve-fitting; a rising straight line could have been more reasonably employed.

TABLE IX

MEASURE OF THE UNRELIABILITY OF THE TOTAL
INSPECTION LABOR COST FUNCTION*

(Standard error of difference between partial
regression curve at specified values of
output and partial regression curve
at average value of output)

Output (Unit \$1,000 in O.W.V.)	Estimated Cost		Standard Error of Deviations in Col. 3†
	Reading from Curve	Deviations from Reading at Mean Output†	
(1)	(2)	(3)	(4)
5.....	\$ 80	\$ -168	\$64.41
10.....	140	-108	70.45
15.....	182	- 66	86.08
20.....	202	- 45	49.46
25.....	219	- 29	24.37
30.....	249	1	5.93
35.....	296	48	25.77
40.....	323	75	41.55
45.....	329	81	62.02
50.....	359	111	-----

*The method of computing this standard error is described in Ezekiel, Methods of Correlation Analysis, p. 384. The formulas are provisional and give only a rough approximation.

† Departures of curve readings from the curve value \$248, which corresponds to the mean of output (\$29,720).

‡ Standard error refers to departures described in the preceding footnote.

A study of the reliability of the regression curve for output (Table IX) indicates that a straight line was as flexible a function as the data justified. The undulations of the central section of the curve fitted to these data were clearly not statistically significant.

Measurements of the degree of success attained in this analysis of total inspection cost behavior are not encouraging. The standard error of estimate was found to be \$79.59, as compared with a standard deviation of \$87.72, while the index of multiple correlation stood at .456 and the index of multiple determination at .208. Evidently estimates made from these curves must be supplemented by allowance for other causal facts not included in this analysis.

Yard Labor Cost

The upper chart of Figure 12 shows that the output regression curve of total yard labor cost rose continuously from an apparently irreducible minimum of about \$600 to an asymptotic top of about \$1,900. This behavior accorded well with expectations except that the flexibility of yard labor in adjusting itself to short-period changes in the level of production was rather surprising. It can be seen from the second chart of Figure 12 that increasing the size of the production order had an uncertain effect upon yard labor cost. The net regression curve at first rose, then declined, with wide see-sawing of the line of averages. The regression curve for the third chart shows that a striking increase in the total of yard labor cost accompanies a reduction in the level of output from that of the previous period. And the bottom chart of Figure 12 shows that no relationship between yard labor cost and lumber receipts was discovered.

The adjusted standard error of the estimates of total yard labor cost made on the basis of these net regression curves was \$392.46. This indicates that the chances are only one in three that the estimate of this cost will miss the actual cost by more than about \$400. The index of multiple correlation (.689) showed that estimates of cost based upon the regression curves correlated only fairly well with the actual cost figures. The index of total determination was found to be .475, indicating that only a relatively small proportion of the variance of yard labor cost was accounted for by the independent factors used in this study. Estimates made on the basis of these curves should, therefore, be used cautiously in making managerial decisions and should be supplemented, where possible, with estimates of the effects of factors not included in this study.

PARTIAL REGRESSION CURVES OF YARD LABOR TOTAL COST FOR FOUR INDEPENDENT FACTORS: OUTPUT (X_2), SIZE OF PRODUCTION ORDER (X_3), INCREASE IN PRODUCTION (X_5), AND LUMBER RECEIPTS (X_9).

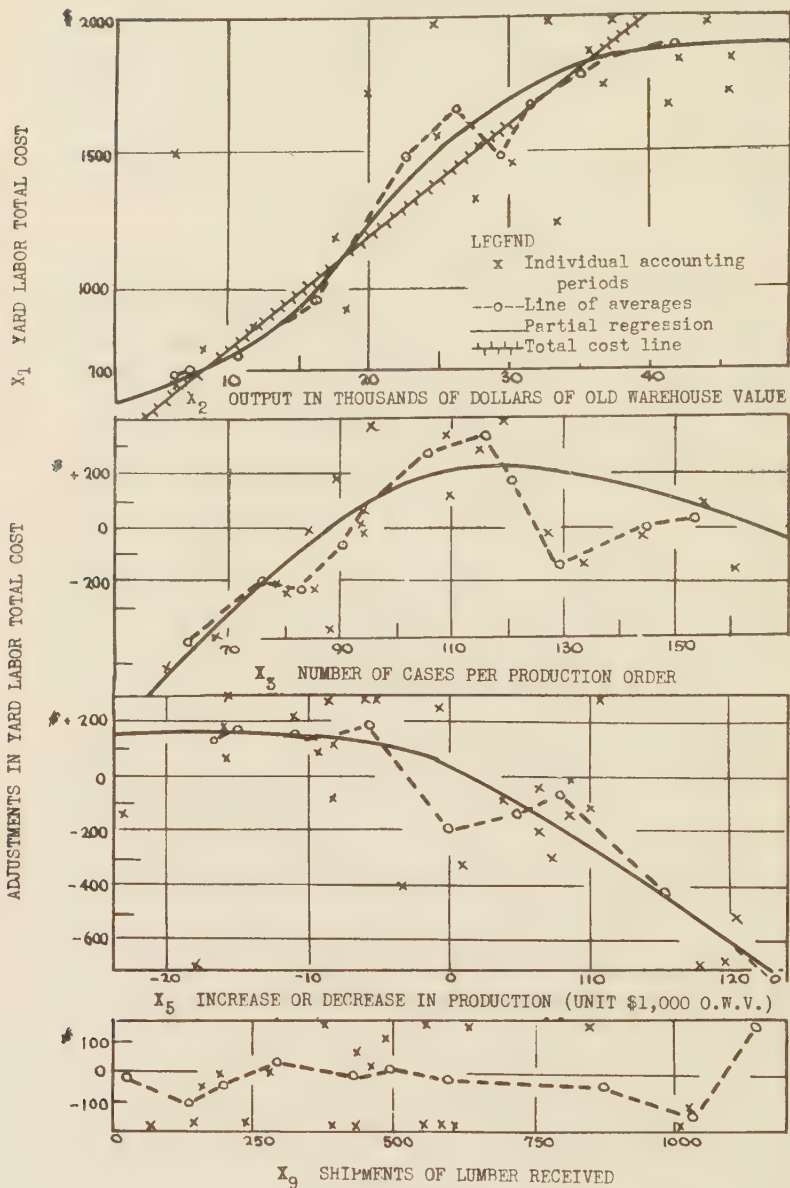


FIG. 12

The computed standard error of the departures of the output curve from its value at the mean level of output are shown in Table X. Examination of this table led to the conclusion that between \$5,000 and \$30,000 output the curve is significantly different from its mean value, but that little significance can be attached to it beyond \$35,000 output.

TABLE X

MEASURE OF THE UNRELIABILITY OF TOTAL
YARD LABOR COST FUNCTION*

(Standard error of difference between partial
regression curve at specified values of
output and partial regression curve
at average value of output)

Output (Unit \$1,000 in O.W.V.)	Estimated Cost		Standard Error of Deviations in Col. 3†
	Reading from Curve	Deviations from Reading at Mean Output‡	
(1)	(2)	(3)	(4)
5.....	\$ 610	\$-1,072	\$390.09
10.....	766	- 916	348.41
15.....	964	- 718	368.68
20.....	1,236	- 446	244.62
25.....	1,473	- 209	147.74
30.....	1,649	- 33	32.95
35.....	1,788	106	139.65
40.....	1,779	197	251.58
45.....	1,892	210	306.69
50.....	1,899	217	-----

*The method of computing this standard error is described in Ezekiel, Methods of Correlation Analysis, p. 384. The formulas are provisional and give only a rough approximation.

†Departures of curve readings from the curve value \$1,682, which corresponds to the mean of output (\$29,720).

‡Standard error refers to departures described in the preceding footnote.

Indirect Material Cost

From Figure 13 it appears that indirect material cost per unit first declines and then rises as the rate of output is increased. This behavior may be caused by the composite character

PARTIAL REGRESSION CURVES FOR AVERAGE INDIRECT MATERIAL COST PER UNIT WITH FIVE INDEPENDENT FACTORS: OUTPUT (X_2), SIZE OF PRODUCTION ORDER (X_3), NEW STYLES (X_4), PRODUCTION INCREASES (X_5), AND QUALITY OF MATERIALS (X_8).

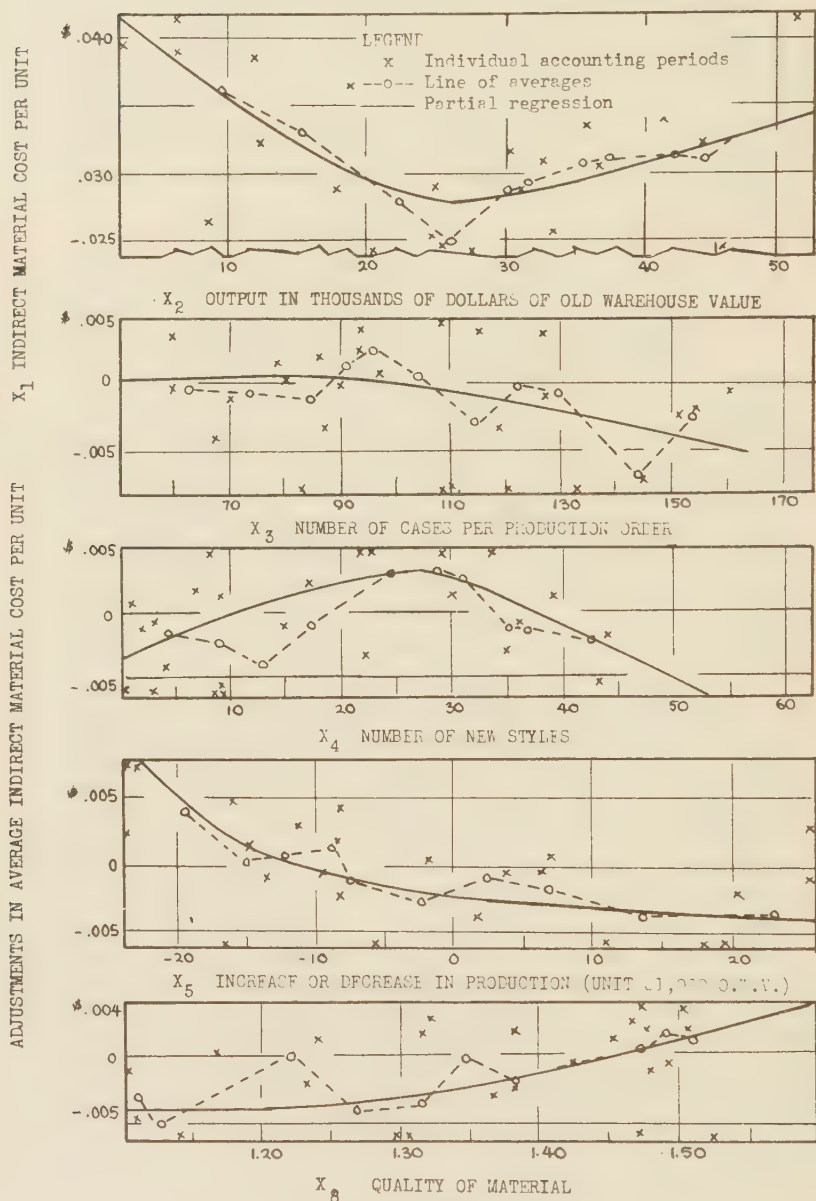


Fig. 13

of this cost, which is made up of sandpaper, tools, and finishing materials. Per-unit tool cost should reasonably decline since it is comparatively fixed in total. Sandpaper cost, on the other hand, probably rises with heavy production because of pecuniary diseconomies, substitution of wasteful (i.e., wasteful of sandpaper) machine processes for hand sanding, and operation of these machines at uneconomical speeds. Finishing material cost probably behaves similarly because of the wastes of machine processes and of high-speed production.

TABLE XI

MEASURE OF THE UNRELIABILITY OF THE AVERAGE
INDIRECT MATERIALS COST FUNCTION*

(Standard error of difference between partial
regression curve at specified values of
output and partial regression curve
at average value of output)

Output Unit \$1,000 in O.W.V.)	Estimated Cost		Standard Error of Deviations in Col. 3:
	Reading from Curve	Deviations from Reading at Mean Output	
(1)	(2)	(3)	(4)
5.....	\$.03937	\$.01095	\$.00539
10.....	.03561	.00719	.00482
15.....	.03224	.00382	.00510
20.....	.02972	.00140	.00338
25.....	.02820	.00022	.00204
30.....	.02844	.00002	.00049
35.....	.02958	.00116	.00216
40.....	.03083	.00241	.00348
45.....	.03224	.00382	.00424
50.....	.03381	.00539	.00847

*The method of computing this standard error is described in *Biometrical Methods of Correlation Analysis*, p. 384. The formulas are provisional and give only a rough approximation.

Departures of curve readings from the curve value \$.02842, which corresponds to the mean of output (\$29,720).

Standard error refers to departures described in the preceding footnote.

Certain savings, particularly in finishing materials, appear to accompany large manufacturing lots. This expectation is substantiated by the second chart of Figure 13. Increases in the number of new styles are associated first with an increase in indirect material cost, and later with a decrease, which decline is poorly supported both rationally or empirically. The effect of sharp increases and decreases which is indicated by the fourth curve is entirely reasonable in view of the inflexibility of certain elements of indirect material cost. The rise of the regression curve for quality of materials is also in full accord with expectations, since more expensive finishing materials and larger proportions of sanded surface usually accompany a higher quality of product.

The standard error of departures of the output regression curve from its value at the mean level of output indicates that little confidence can be placed in the major portion of this curve (see Table XI). Moreover, the adjusted index of multiple correlation was found to have a value of only .343, which does not indicate sufficiently high correlation between estimated indirect material cost and actual cost to justify great confidence in the results of the analysis. Apparently some factors which have an important influence upon indirect material cost have been omitted. Hence, only limited reliance can be placed in these regression curves despite their apparent reasonableness.

Power Cost

Figure 14 shows the regression curves for average purchased power cost. From the top section of this figure it can be seen that this cost falls as output increases. This behavior is altogether reasonable since the public utility rate structure penalizes the firm for idleness, and since efficiency in the use of power is increased and transmission wastes are reduced by full operation. The middle chart of Figure 14 shows an entirely logical decline of power cost as the size of manufacturing lot is increased. The lower chart is also sensible, for it indicates

PARTIAL REGRESSION CURVES FOR AVERAGE PURCHASED POWER COST PER UNIT
WITH THREE INDEPENDENT FACTORS: OUTPUT (X_2), SIZE OF PRO-
DUCTION ORDER (X_3), AND NEW STYLES (X_4).

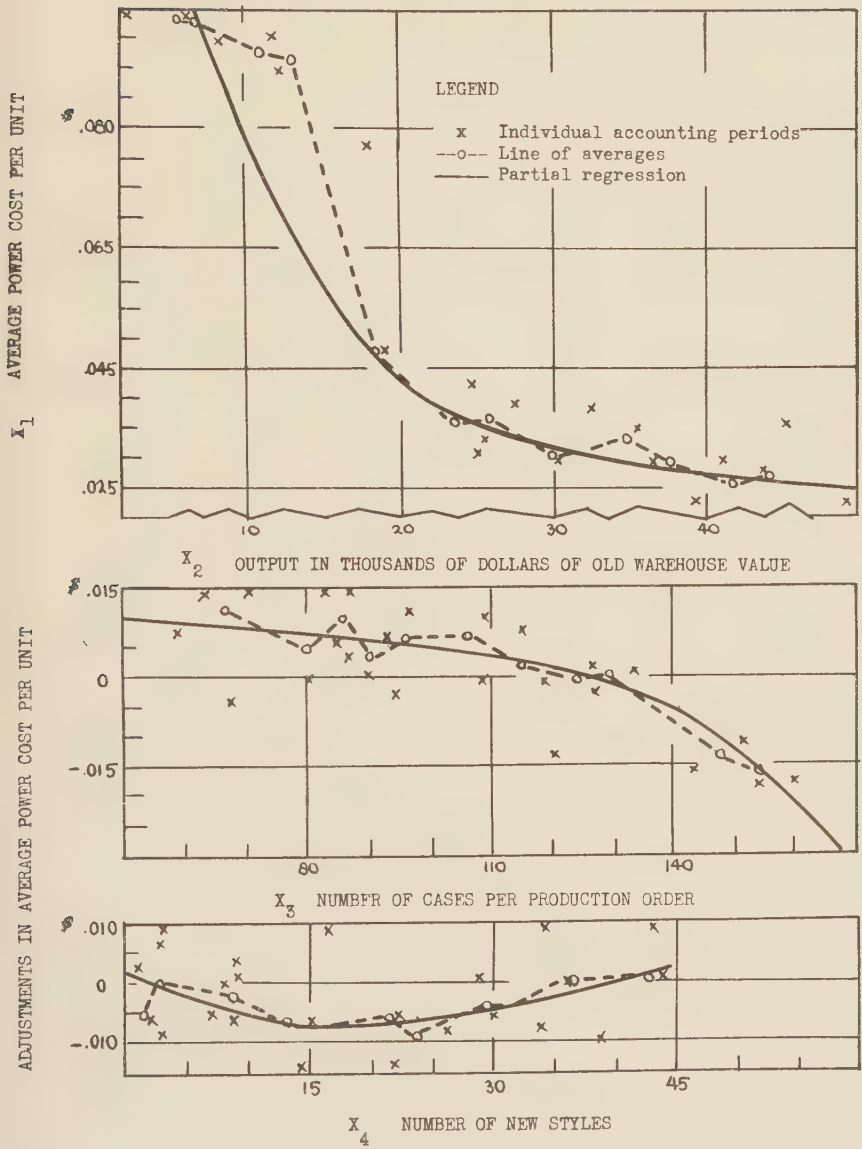


FIG. 14

that the number of new styles has no significant effect upon power cost per unit of output.

TABLE XII

MEASURE OF THE UNRELIABILITY OF THE AVERAGE
POWER COST FUNCTION*

(Standard error of difference between partial
regression curve at specified values of
output and partial regression curve
at average value of output)

Output (Unit \$1,000 in O.W.V.)	Estimated Cost		Standard Error of Deviations in Col. 3†
	Reading from Curve	Deviations from Reading at Mean Output†	
(1)	(2)	(3)	(4)
10.....	\$.0843	\$.0527	\$.0517
15.....	.0562	.0246	.0547
20.....	.0426	.0110	.0363
25.....	.0354	.0038	.0219
30.....	.0313	- .0003	.0053
35.....	.0287	- .0029	.0232
40.....	.0271	- .0045	.0373
45.....	.0257	- .0059	.0455
50.....	.0244	- .0072	.0908

*The method of computing this standard error is described in Ezekiel, Methods of Correlation Analysis, p. 384. The formulas are provisional and give only a rough approximation.

†Departures of curve readings from the curve value \$0.0316, which corresponds to the mean of output (\$29,720).

‡Standard error refers to departures described in the preceding footnote.

From Table XII we see that small reliance in the departures of the regression curve for output from its value at the mean level of output is indicated by the standard error of this curve. The three regression curves taken together, however, account for a fair proportion of the variance of average power cost. The index of multiple correlation was found to be .637, and the standard error of estimate \$0.062 (see Table II).

Maintenance Cost

Average maintenance cost declines as output increases, as can be seen from the top chart of Figure 15. This behavior seems to be partially due to the fact that some of the wear and tear which causes maintenance is a function of time rather than of output.³ But it is probably largely caused by the policy of this firm of using postponable maintenance as a sort of unemployment benefit.⁴ In the middle chart of Figure 15 a slight decline of maintenance cost as a result of increasing the size of the production order is indicated. The third chart shows that quick changes in the rate of production have no significant effect upon maintenance cost.

By means of these three regression curves average maintenance cost could be estimated with a high degree of accuracy, as is shown by the small size of the standard error of estimate, \$0.0296, as compared with a standard deviation of \$0.076. The correlation between maintenance cost and the three independent variables was high. The adjusted index of multiple correlation was computed as .924. This cost item ranks high among the average cost items with respect to the size of its adjusted correlations index (see Table II).⁵ Hence, these regression curves appear to represent a very satisfactory description of the behavior of this cost item.

Three cost items--combined indirect cost, direct material cost, and power and watch labor cost--were also analyzed. Because of limited space the results are not presented in this abstract but may be found in the author's dissertation,⁶ where those cost

³When the constant total of this portion of maintenance cost is spread over a larger output, the cost per unit declines.

⁴This practice results in incorrect allocation of cost between periods. Part of this cost should properly have been charged to past periods when the machines were actually worn out. Part should be charged to future periods when the working force which is held together in this manner will begin to function again with smaller initial recruiting cost than otherwise would be the case.

⁵The meaning of correlation indexes obtained from cost data in the form of averages per unit of output is not parallel to, or exactly comparable with, these secured from data in the form of totals.

⁶"A Statistical Examination of the Behavior of Average and Marginal Costs," chap. vii and Appendix C.

PARTIAL REGRESSION CURVES FOR AVERAGE MAINTENANCE COST PER UNIT WITH THREE
INDEPENDENT VARIABLES: OUTPUT ($\bar{x}_2$), SIZE OF PRODUCTION ORDER ($\bar{x}_3$),
AND PRODUCTION INCREASES ($\bar{x}_5$).

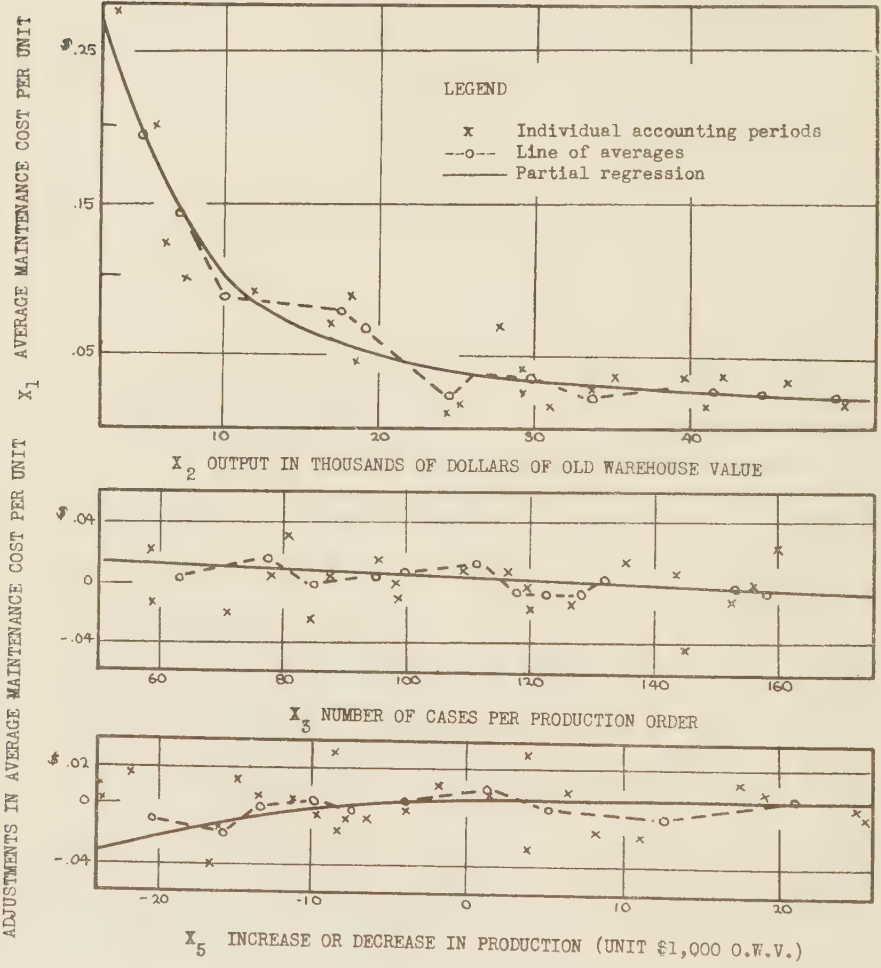


FIG 15

items which are presented in this abstract are also discussed more fully. Several components of the long-period cost of a finance company were also studied. These, too, could not be discussed here, but are found in Appendix E of the author's dissertation.

APPENDIX B

BEHAVIOR OF COMPONENTS OF MARGINAL COMBINED COST

Introduction

Before undertaking a discussion of the components of marginal combined cost it is important to define the meaning of this phrase. The direct labor component, for example, is not the partial increment of cost associated with increasing the output by varying labor cost alone. Instead, it is the increment in direct labor cost associated with an increase in output of one unit (\$1.00 O.W.V.) when such an increment of output is obtained in the customary way, i.e., with the other variable cost elements increasing as they typically do. As was indicated in an earlier chapter, such a study is of use only when, for given operating conditions, the combination of cost constituents is fixed within rather narrow limits.

Since this condition appears to be approximated in the firm studied, some practical justification for a study such as this may be found in the expectation that a knowledge of the behavior of the components of marginal cost may increase the accuracy of forecasting and allocating combined marginal cost. The accuracy of forecasts may be enhanced by such a study partly because specific allowance can then be made for changes in conditions which affect a particular cost component. For example, an expected increase of 20 per cent in the wage rates of yard labor could be reflected fairly accurately by a 20 per cent increase of the estimate of the yard labor component of marginal cost, but could not be allowed for at all in an estimate of marginal combined cost without a knowledge of the relative size of the yard labor cost constituent.

A second way in which study of the behavior of these constituent elements may enhance accuracy is by providing a rough check upon the estimate of combined marginal cost. This can be accomplished by building up the combined marginal cost function by summation of its components. The estimate of each component

of marginal cost is arrived at independently from the recorded totals of this particular type of expense. If the sum of these constituents equals the marginal cost obtained by differentiating total combined cost, faith in the accuracy of derivation is buttressed.¹

In addition to the hope that a knowledge of the individual components may increase the accuracy of the estimate of marginal combined cost there is also a possibility that such information may improve the precision with which this estimated cost is allocated to particular items of product. Without such knowledge the average of combined marginal cost of all products is allocated to a particular product on the basis of our common denominator of output (old warehouse value). This procedure assumes, first, that old warehouse value is a satisfactory measure of output; second, that the components of marginal cost behave in approximately the same fashion regardless of the product being manufactured; and, third, that the proportion in which these various components are combined remains approximately constant for all articles of product.

It is with this third assumption that we are particularly concerned here, for the first two form the basis of the proposed refined method of allocation as well as for the average method. The assumption that articles do not differ in the proportion in which their several cost components are combined is patently incorrect. Certain types of bedroom furniture, for example, employ an inordinately large proportion of indirect materials and indirect labor.

A suggested alternative procedure for allocation is to build up an estimate of the combined marginal cost of an article as the sum of the marginal components. This could be accomplished as follows:

1. The standard average cost of an article is broken down into its components.
2. The normal operating conditions for which this standard was set are specified.
3. The deflated average cost of each component is determined from the regression curves for these specified normal conditions.

¹In this particular study the check could not be used since not all the elements of combined cost were studied.

4. The deflated marginal cost of each component is ascertained from the regression curves and from estimated marginal cost functions for the operating conditions under which the article is expected to be produced.
5. The ratio of the marginal cost determined in (4) to the deflated average cost found in (3) is obtained.
6. These ratios are used to weight the respective components of standard average cost in (1), which are then summed to give the estimated marginal combined cost.

There are several important assumptions underlying such a procedure:

1. Standard average cost and its components are satisfactory.
2. Conditions for which such standards are computed can be specified.
3. The ratios of marginal cost under any given conditions to average cost under fixed normal or standard conditions are approximately the same for all articles. If the marginal cost of components is quite variable with output, this assumption would probably necessitate that the combination of cost elements at any given output be fairly stable regardless of the articles composing that output. Even if this were true, it is conceivable that these ratios might vary considerably for different articles.
4. For any given size of output composed of specified articles the proportion of cost constituents is fixed within rather narrow limits.
5. Old warehouse value measures output accurately.
6. Curve-fitting errors of the estimates of the marginal cost of components are somewhat compensating, so that accuracy is not destroyed by cumulative errors.

In determining which components of marginal cost were to be studied an effort was made to choose, first, cost items which would be of greatest interest to management; and, second, costs whose behavior would be difficult to estimate short of a systematic analysis--combined indirect labor, inspection labor, yard labor, power and watch labor, indirect materials, power, maintenance, direct materials, direct labor, and combined indirect cost. The methods employed in studying these costs were similar to those used for combined cost. Corrected data for most components were studied both in the form of totals and in the form of averages.²

²To the total cost data of any particular component a function (usually a straight line) was fitted freehand, then converted, by the method of selected points, into an equation which

A quick grasp of the findings of this phase of the study can be had by examining Tables XIII-XXII. The results obtained by fitting straight lines to data in the form of totals are exhibited in Table XIII, where the estimate of constant marginal cost together with the total cost equation from which it was derived are presented. The findings obtained by fitting curvilinear functions to average and to total cost data are shown separately for each cost item in Tables XV-XXII. Each of these tables shows estimates of the average, total, and marginal cost of the component at regularly spaced output levels. Below the table are presented the equations from which the estimates were obtained.³ All estimates of marginal cost are expressed as the increment in deflated dollar cost (as of January 1, 1932) which accompanies an increment in output of one dollar of old warehouse value.

Before considering the findings it will be well to call attention to certain factors which affect their reliability. In general, the trustworthiness of the estimates of a particular component of marginal cost depends upon:

1. The form in which the data were studied. Totals appear to give more reliable results than averages since there is less danger of fitting spuriously complex functions to total cost observations.
2. The degree of faithfulness with which the graphically fitted average or total cost curve represents the data. Often the scatter is great and the function too flexible.
3. The exactness with which the graphic curve is described by the average or total cost equation. For straight lines this source of error is not important. But when the function is a complex curve, then an equation obtained by the method of selected points does not always fit the curve closely at all points.

was differentiated to obtain marginal cost. Data in the form of averages were studied in a similar fashion. To a curve, drawn freehand, an equation was fitted by the method of selected points. This equation was transformed to a total cost equation and differentiated to obtain the marginal cost function.

³The symbols used in these equations have the following meanings:

- X_2 , output in terms of thousands of dollars of old warehouse value
- X_T , total cost in thousands of deflated dollars
- X_A , average deflated cost per unit of output, i.e., per dollar of old warehouse value
- X_M , marginal cost (unit, one deflated dollar) of an increase in output of one dollar of old warehouse value

TABLE XIII

ROUGH CONSTANT ESTIMATE OF MARGINAL COST
FOR SEVERAL ITEMS OF COST

(These components of combined cost have been adjusted
for price changes, output lag, and for the
effects of independent variables
of the correlation analysis)

Cost Item	Total Cost Equation ($X = \$1,000$ O.W.V.)	Marginal Cost Estimate (Per Dollar O.W.V.)
<u>Components:</u>		
Indirect labor.....	$\underline{Y} = \$ 3,700 + 24 \underline{X}$	\$0.024
Inspection labor.....	$\underline{Y} = 68 + 6.4 \underline{X}$	0.0064
Yard labor.....	$\underline{Y} = 390 + 40 \underline{X}$	0.04
Power and watch labor.	$\underline{Y} = 640 + 13 \underline{X}$	0.013
Power cost.....	$\underline{Y} = 548 + 13.8 \underline{X}$	0.0138
Maintenance.....	$\underline{Y} = 964 + 2.4 \underline{X}$	0.0024
Direct labor.....	$\underline{Y} = 2,700 + 505 \underline{X}$	0.505
All indirect cost.....	$\underline{Y} = 7,040 + 71 \underline{X}$	0.071
<u>Combined cost:</u>		
Wholly uncorrected....	$\underline{Y} = 11,800 + 1,220 \underline{X}$	1.22
Corrected for price and lag alone.....	$\underline{Y} = 12,400 + 1,090 \underline{X}$	1.09
Corrected by multiple correlation analyses only.....	$\underline{Y} = 17,900 + 790 \underline{X}$	0.79
Fully corrected.....	$\underline{Y} = 13,360 + 1,115.6 \underline{X}$	1.116

Direct Labor Cost

The best estimate of the direct labor component of marginal cost which could be made from these data is that it declines at first as output is increased up to about 25 per cent capacity, then remains approximately constant over the remaining observed range of output. The estimated constant cost, obtained by fitting a straight line to the total cost data, is \$0.505. This estimate was supplemented by a curvilinear incremental cost function obtained by the graphic method from data in the form of averages. The shape and position of this marginal cost curve substantiate, in general, the estimate of constant incremental cost, for the curve departs significantly from this horizontal line only for

small outputs. In this latter region the curve seems to indicate, as did the distribution of the total cost data, that marginal cost declines as output is increased from very low levels.

Except for this initial falling phase of the marginal cost curve only its general level appears to be significant. The minor undulations seem to be due to the fact that slight changes in the slope of the average cost curve bring pronounced and, according to the distribution of the total cost function, unwarranted fluctuations in the marginal cost function.

TABLE XIV
AVERAGE AND MARGINAL COST OF DIRECT LABOR

(Selected items from determination by
the graphic method)

Output* (Unit \$1,000 O.W.V.)	Average Cost†	Moving Average of Marginal Cost‡
5.....	\$1.0700	\$.8288
10.....	0.8140	.4480
15.....	0.6720	.3885
20.....	0.6130	.4796
25.....	0.5967	.5510
30.....	0.5883	.5402
35.....	0.5800	.5236
40.....	0.5717	.5061
45.....	0.5633	.4903

* Items selected at intervals of \$5,000 O.W.V. from a table computed at intervals of \$1,000 O.W.V.

† Direct labor cost per dollar of old warehouse value, read from net regression curve of average labor cost on output.

‡ Five-item moving average of first differences of the total cost series obtained by transformation of reading from the average cost curve, cost increment corresponding to an output increment of \$1.00 O.W.V.

Direct Materials Cost

Direct materials include all supplies used in the manufacture of furniture which can be assigned directly to particular lots or product. The most important components of direct materials are lumber, veneer, and glue. The average, total, and marginal

cost of direct materials for various levels of output, as determined from average cost observations by the graphic-derivative method, are summarized in Table XV. From this table it appears that marginal cost first declines slightly, then rises continuously as output is increased, exceeding average cost at all levels beyond \$20,000 output and ranging between about \$0.60 and \$1.35.

TABLE XV
AVERAGE, TOTAL, AND MARGINAL COST OF DIRECT MATERIALS

(Determined by the graphic-derivative method
at regular intervals of output)

Output (Unit \$1,000 O.W.V.)	Average Cost (Per Dollar of O.W.V.)*	Total Cost†	Marginal Cost‡
5.....	\$.657	\$ 3,285	\$0.641
10.....	.646	6,465	0.629
15.....	.641	9,615	0.639
20.....	.643	12,860	0.671
25.....	.654	16,350	0.726
30.....	.673	20,190	0.805
35.....	.698	24,430	0.905
40.....	.732	29,280	1.028
45.....	.773	34,785	1.174
50.....	.821	41,050	1.343
10.31....	----	-----	Minimum

* Average cost equation: $\underline{X}_A = .677 - .00467\underline{X} + .000151\underline{X}^2$.

† Total cost equation: $\underline{X}_T = .677\underline{X} - .00467\underline{X}^2 + .000151\underline{X}^3$.

‡ Marginal cost equation: $\underline{X}_M = .677 - .00934\underline{X} + .000453\underline{X}^2$.

However, the degree of reliance which can be placed in this schedule of marginal cost estimates probably is not great, for the average cost curve from which it was derived is of limited trustworthiness.⁴ An estimate of about \$0.65 for constant marginal cost is probably safer than these curve estimates.

⁴For discussion of the average cost behavior of this cost item see the author's dissertation, *op. cit.*, chap. vii and Appendix C.

Combined Indirect Cost

Combined indirect cost included all overhead expenses investigated in this study, together with a few minor items not analyzed in detail. Probably the most serviceable estimate of marginal indirect cost is that it is constant at about \$0.071 (Table XIII). Besides this constant value a curvilinear marginal cost function was also computed. The equation for this curve and for the average and total cost curves from which it was derived are shown and evaluated in Table XVI. Examination of the marginal

TABLE XVI

AVERAGE, TOTAL, AND MARGINAL ESTIMATES OF COMBINED INDIRECT COST

(Determined by the graphic-derivative method
at regular intervals of output)

Output (Unit \$1,000 O.W.V.)	Average Cost (Per Dollar of O.W.V.)*	Total Cost†	Marginal Cost‡
5.....	\$1.260	\$ 6,300	\$0.743
10.....	0.769	7,690	0.234
15.....	0.554	8,310	- 0.115
20.....	0.440	8,800	- 0.306
25.....	0.371	9,275	- 0.337
30.....	0.323	9,690	- 0.210
35.....	0.292	10,220	0.077
40.....	0.282	11,280	0.522
45.....	0.300	13,500	1.127
23.64....	-----	-----	Minimum

* Average cost equation: $\underline{X}_A = 1.410 - .0747\underline{X}_2 + .00106\underline{X}_2^2$.

† Total cost equation: $\underline{X}_T = 1.410\underline{X} - .0747\underline{X}_2^2 + .00106\underline{X}_2^3$.

‡ Marginal cost equation: $\underline{X}_M = 1.410 - .1494\underline{X}_2 + .00318\underline{X}_2^2$.

cost magnitudes in this table shows that marginal cost is great for small outputs, then falls sharply to the right as output is increased, and finally rises again for large outputs.

The reliance which can be placed in the curve for marginal cost is not great because the data were analyzed in the form of averages rather than in the form of totals, and because the average

cost equation did not fit its curve well for outputs in excess of \$30,000. Possibly the initial falling phase is significant, for the average cost curve is well established and faithfully reflected by its equation in this range. Moreover, this behavior is a reasonable accompaniment of the disorganization costs of operating considerably below the designed scale. Thus, the best estimate which can be made from this study is that differential indirect cost is constant at about \$0.071 for outputs in excess of 30 per cent capacity, and that it probably falls from a much higher level for small outputs before settling to this constant level at about 30 per cent capacity (\$15,000 O.W.V.).

Combined Indirect Labor Costs

Combined indirect labor cost is composed of the wage costs of the following indirect departments: inspection, power and watch, stockrooms, yard, and supervision. Two estimates were made of the magnitude of this important component. The first, which was arrived at by fitting a straight line to total cost of observations, is a constant--\$0.024 (Table XIII). This estimate is probably most nearly correct for the middle range of output. The second estimate is a parabolic marginal cost function derived from a curve fitted to average cost observations. Its equation is given and evaluated in Table XVII. From this table it appears that marginal cost increases at both extremes of the output range--a behavior pattern which appears reasonable in the light of the disorganization costs which are likely to accompany any substantial departure from the operating level for which the plant was designed. Limited confidence, however, can be placed in the undulations of this marginal cost curve.⁵ Probably all that can be reliably stated about this component is that its magnitude declines from high values for small outputs to a level of about \$0.024 for production in excess of about 50 per cent capacity.

⁵In the first place, data in the form of averages rather than in the preferred form of totals were used to determine this curve; second, the scatter of observations about the average cost curve was so great that its departures from the horizontal appeared to have limited statistical significance beyond \$30,000 output (Table VIII); third, the fit of the average cost equation to this graphically determined curve was not good at large outputs. Consequently the increase in marginal cost estimates for these higher production levels is probably incorrect.

TABLE XVII

AVERAGE, TOTAL, AND MARGINAL COST OF INDIRECT LABOR

(Determined by the graphic-derivative method
at regular intervals of output)

Output (Unit \$1,000 O.W.V.)	Average Cost (Per Dollar of O.W.V.)*	Total Cost†	Marginal Cost‡
10.....	\$.3602	\$3,590	\$.1919
15.....	.2665	3,998	.1050
20.....	.2104	4,208	.0458
25.....	.1735	4,338	.0144
30.....	.1472	4,398	.0107
35.....	.1291	4,508	.0348
40.....	.1198	4,792	.0866
45.....	.1202	5,409	.1662
50.....	.1240	6,200	.2735
28.162...	-----	-----	Minimum

* Average cost equation: $\underline{X}_A = .4490 - .01563\underline{X}_2 + .000185\underline{X}_2^2$.† Total cost equation: $\underline{X}_T = .4490\underline{X}_2 - .01563\underline{X}_2^2 + .000185\underline{X}_2^3$.‡ Marginal cost equation: $\underline{X}_M = .4490 - .03126\underline{X}_2 + .000555\underline{X}_2^2$.Inspection Labor Cost

As an experiment in methodology, inspection labor cost and two other components--yard labor and power and watch labor--were kept in terms of totals rather than in terms of averages per unit of output throughout the multiple correlation analysis. This experiment substantiated the conclusions reached from the analysis of combined cost, namely, that marginal cost could be derived more economically and more reliably when data were analyzed in the form of totals than when studied in the form of averages.

Two types of total cost functions were fitted visually to these adjusted total cost observation residuals--a straight line and a curve. From the straight line an estimate of constant marginal cost of \$0.0064 was derived (Table XIII). From the curve a parabolic marginal cost function was computed. Its equation and the total cost equation from which it was derived are given and evaluated in Table XVIII. At most, only the general level of this

marginal cost curve is significant,⁶ and probably the best estimate to be made from the data is that marginal cost is constant at about \$.0064 regardless of the rate of output.

TABLE XVIII

TOTAL, AVERAGE, AND MARGINAL COST OF INSPECTION LABOR

(Determined by the graphic-derivative method
at regular intervals of output)

Output (Unit \$1,000 O.W.V.)	Total Cost*	Average Cost (Per Dollar of O.W.V.)	Marginal Cost†
5.....	\$ 80	\$.01600	\$ -.01770
10.....	140	.01400	-.00817
15.....	182	.01213	-.00110
20.....	202	.01013	.00405
25.....	219	.00876	.00710
30.....	261	.00870	.00805
35.....	296	.00846	.00690
40.....	323	.00808	.00365
45.....	345	.00766	-.00170
50.....	359	.00718	-.00915
29.76....	---	-----	Maximum

*Total cost equation:

$$\underline{X}_T = 385.25 - 29.15\underline{X}_2 + 1.25\underline{X}_2^2 - .014\underline{X}_2^3.$$

†Marginal cost equation: $\underline{X}_M = 29.15 + 2.5\underline{X}_2 - .042\underline{X}_2^2$.

Yard Labor Cost

Another important type of indirect labor in a furniture factory is yard labor. In this cost are summarized the wages of the men whose functions are (1) to unload incoming lumber, (2) to stack it for storage, and (3) to transport it to the kilns and to the factory. Like inspection labor, yard labor cost was kept in the form of totals throughout the multiple correlation analysis. To the adjusted observations in the output regression chart of

⁶The sample was small and was widely scattered in a distribution which seemed to be linear. The standard error of this total cost curve (Table IX) indicated that its convolutions were not statistically significant.

this analysis two types of functions were fitted--a straight line and a cubic parabola. From the straight line marginal cost was estimated to be constant at \$0.04 per dollar of old warehouse value (Table XIII). However, there may be grounds for believing that the cost rises from about \$0.03 for small outputs to about \$0.05 for medium outputs, and then declines somewhat for high rates of pro-

TABLE XIX

TOTAL, AVERAGE, AND MARGINAL COST OF YARD LABOR

(Determined by the graphic-derivative method
at regular intervals of output)

Output (Unit \$1,000 O.W.V.)	Total Cost*	Average Cost (Per Dollar of O.W.V.)	Marginal Cost†
5.....	\$ 610	\$.125	\$.03105
10.....	766	.076	.04232
15.....	964	.064	.04794
20.....	1,236	.062	.04792
25.....	1,473	.059	.04224
30.....	1,649	.055	.03092
35.....	1,718	.051	.01394
40.....	1,779	.044	- .00868
45.....	1,755	.039	- .03696
50.....	1,700	.034	- .10784
17.48....	-----	----	Maximum

* Total cost equation:

$$\underline{X}_T = 464.98 + 14.119\underline{X}_2 + 1.975\underline{X}_2^2 - .03767\underline{X}_2^3.$$

† Marginal cost equation: $\underline{X}_M = 14.119 + 3.95\underline{X}_2 - .11301\underline{X}_2^2.$

duction, for from the cubic parabola of total cost a curvilinear marginal cost function of this general form was derived. Its equation is given and evaluated in Table XIX. Greater reliance, however, can be placed in the estimate of constant marginal cost than in the curve.

Indirect Materials

Indirect materials consist of sandpaper, tools, and finishing supplies. Although they are properly called indirect, the

expenditure for these items fluctuate in rough proportion to output. Table XX shows the estimated marginal cost function derived from a parabolic curve fitted to the adjusted average cost observations. It is clear from this table that marginal cost is approximately constant, although it rises slightly for large rates of output. Moreover, it is significant to note that it does not depart greatly from average cost at any output level although it

TABLE XX
AVERAGE, TOTAL, AND MARGINAL COST OF INDIRECT MATERIALS

(Determined by the graphic-derivative method
at regular intervals of output)

Output (Unit \$1,000 O.W.V.)	Average Cost (Per Dollar of O.W.V.)*	Total Cost†	Marginal Cost‡
5.....	\$.0458	\$ 229	\$.0360
10.....	.0374	374	.0241
15.....	.03224	483	.0200
20.....	.03030	606	.0215
25.....	.02820	705	.0266
30.....	.02850	855	.0331
35.....	.02958	1,035	.0391
40.....	.03100	1,240	.0424
45.....	.03224	1,451	.0459
50.....	.03560	1,780	.0546

*Average cost equation:

$$\underline{X}_A = .0575188 - .00271275\underline{X}_2 + .00007885\underline{X}_2^2.$$

†Total cost equation:

$$\underline{X}_T = .0575188\underline{X}_2 - .00271275\underline{X}_2^2 + .00007885\underline{X}_2^3 - .00000069\underline{X}_2^4.$$

‡Marginal cost equation:

$$\underline{X}_M = .0575188 - .0054255\underline{X}_2 + .00023655\underline{X}_2^2 - .00000276\underline{X}_2^3.$$

may exceed average cost at large outputs. It probably lies between \$0.025 and \$0.05, with its most likely constant value at about \$0.03. The reliability of the curvilinear marginal cost function is probably not great, despite the fact that the average cost equation from which it was derived fits its graphically determined curve well.⁷

⁷There are three serious limitations upon the reliability of this graphic average curve: (1) imperfect correction of the data for price changes; (2) the limited success of the multiple correlation analysis; and (3) the small number of widely scattered

Power Cost

Power cost includes electric current purchased from the public utility to run machines and to light the buildings. The best estimate of its marginal cost behavior which could be made from the data at hand was that incremental cost falls sharply as output is increased from very low levels, then settles to a constant of about \$0.014 for outputs in excess of about \$20,000 O.W.V. In addition to this estimate of constant differential cost, which was based upon a straight line fitted to total cost data (Table XIII),

TABLE XXI

AVERAGE AND MARGINAL COST OF PURCHASED POWER

(Determined by the graphic-derivative method
at regular intervals of output)

Output (Unit \$1,000 O.W.V.)	Average Cost (Per Dollar of O.W.V.)*	Marginal Cost†
5.....	\$.1097	\$.0821
10.....	.0843	.0138
15.....	.0635	.0077
20.....	.0471	- .0565
25.....	.0354	- .0127
30.....	.0279	.0027
35.....	.0251	.0209
40.....	.0271	.0579
45.....	.0329	.1085
50.....	.0435	.1725
23.78.....	-----	Minimum

*Average cost equation: $\bar{X}_A = .1395 - .00642\bar{X} + .000090\bar{X}^2$.

†Marginal cost equation: $\bar{X}_M = .1395 - .01284\bar{X} + .00027\bar{X}^2$.

a curve of marginal cost having considerably less reliability was derived from average cost data by means of the graphic-derivative method.⁸ Regularly spaced values along this curve, together with those for average and total cost, are shown in Table XXI.

observations upon which the curve was based. These limitations are more fully discussed in chap. vii and Appendix C of the author's dissertation, op. cit.

⁸Confidence in this curve of marginal cost is shaken, first, by the fact that data in the form of averages were used; second, by the small size of the sample and the wide scatter of the observations; and, third, by the fact that only a small part of the distorting effect of other variables was removed by the multiple correlation analysis (see Appendix A).

Maintenance Cost

Maintenance cost represents a consolidation of three accounts: (1) maintenance labor, (2) materials for the repair of buildings, and (3) materials for the repair of equipment. Each of these components was corrected separately for price change and time lag before being combined. The marginal cost of maintenance appears to be negligible. Fitting a straight line to data in the form of totals yielded an estimate of constant maintenance cost of about \$.0024 (see Table XIII for equation). But the rate of in-

TABLE XXII

AVERAGE AND MARGINAL COST OF MAINTENANCE

(Determined by the graphic-derivative method
at regular intervals of output)

Output (Unit \$1,000 O.W.V.)	Average Cost (Per Dollar of O.W.V.)*	Marginal Cost†
5.....	\$.1865	\$.1335
10.....	.1376	.0482
15.....	.0969	- .0129
20.....	.0645	- .0493
25.....	.0401	- .0615
30.....	.0238	- .0491
35.....	.0157	- .0124
40.....	.0158	.0488
45.....	.0240	.1344
50.....	.0403	.2445
24.98.....	-----	Minimum

* Average cost equation: $\underline{X}_A = .2434 - .012205\underline{X}_2 + .000162875\underline{X}_2^2$.

† Marginal cost equation: $\underline{X}_M = .2434 - .024410\underline{X}_2 + .00048863\underline{X}_2^2$.

crease of total cost appears to fluctuate somewhat; hence, this estimate is not entirely satisfactory. Actually, marginal cost of maintenance appears to become negative in portions of the middle output range. A schedule of fluctuating marginal cost was therefore also computed. This schedule is shown in Table XXII, together with the equation from which these tabulated values were obtained.

A fair amount of faith may be placed in the curvilinear marginal cost estimates up to about \$35,000 output. Beyond this point, however, the estimates are invalidated by the erroneous departure of the average cost equations from the curve to which they were fitted. Since the form of the data did not appear to be strictly linear, only limited confidence can be placed in the estimate of constant marginal cost.

